

StratLearn:

A general-purpose statistical method
for improved learning under Covariate Shift

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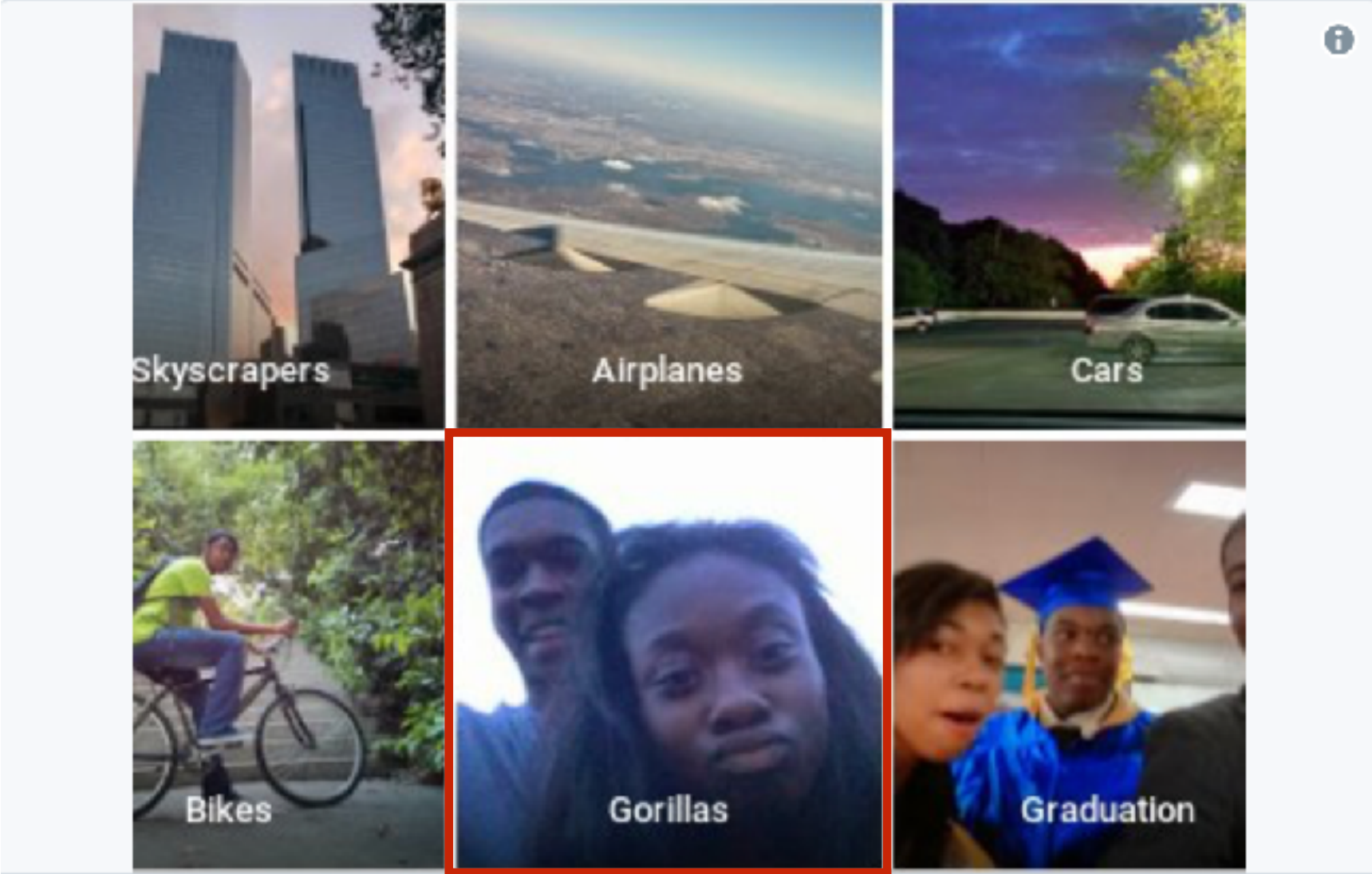
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June 14th 2022

The problem: machine learning classifiers trained on non-representative data generalize poorly.

2015



Jacky lives on @jalcine@playvicious.social now. @jackyalcine

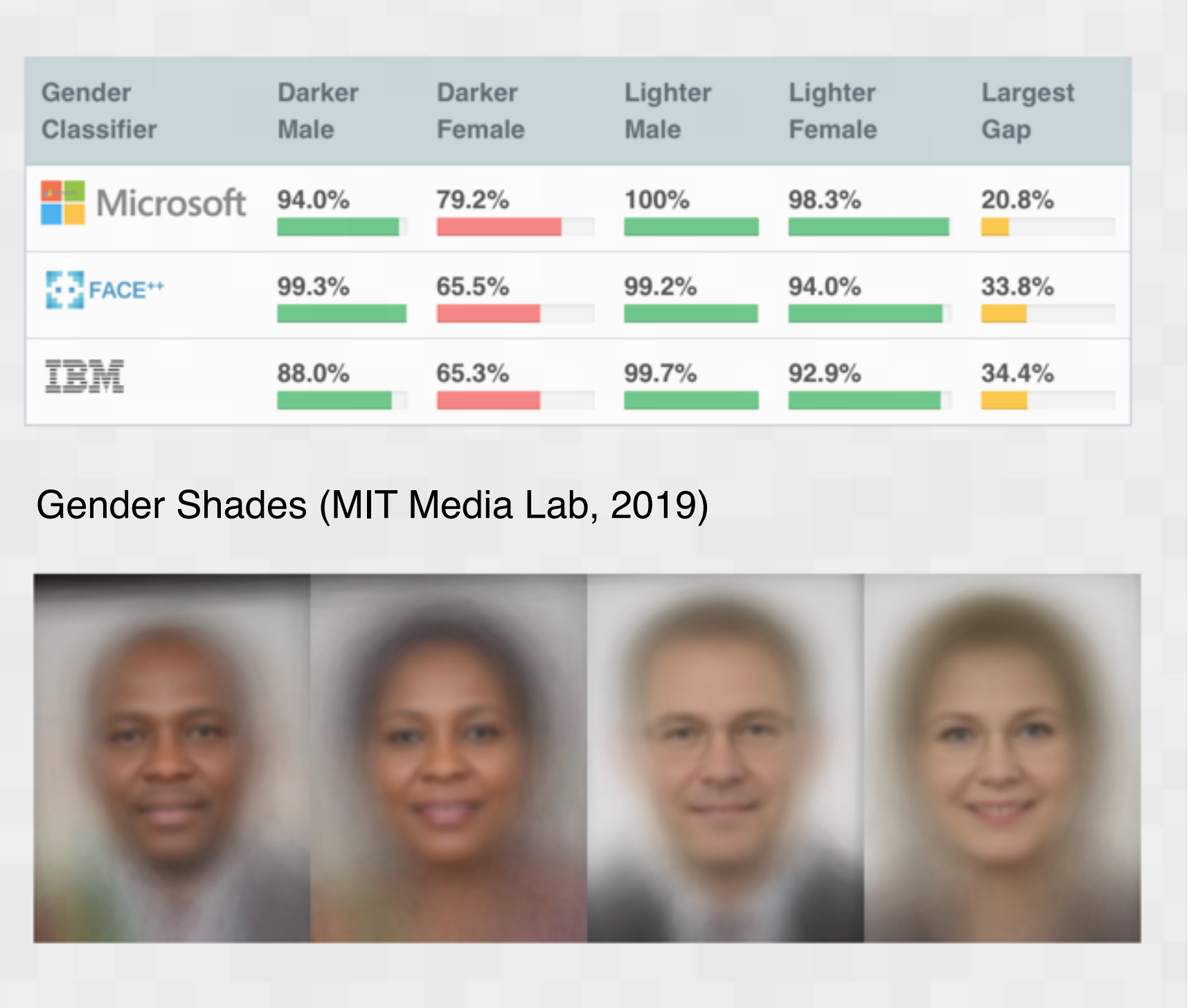
2018

TOM SIMONITE BUSINESS 01.11.2018 07:00 AM

When It Comes to Gorillas, Google Photos Remains Blind

Google promised a fix after its photo-categorization software labeled black people as gorillas in 2015. More than two years later, it hasn't found one.

2019



Poor accuracy in facial recognition for dark skinned females

Cosmology

Incorrect classification of Type Ia vs non-Ia from photometric data leads to cosmological parameters systematic bias.

Radiation
(decelerates)

Dark matter
(decelerates)

Dark energy
(accelerates)

Big Bang
 $t=0$

Inflation
 $t = 10^{-32}$ s
(exponential)

Big Bang
Nucleosynthesis
($t = 3$ mins)

SNIa

End of the visible cosmos
 $t = 380,000$ yrs
 $z=1,100$

$t = 7$ Bn yrs
 $z \sim 0.7$

$t = 13.7$ Bn yrs
 $z=0$

time

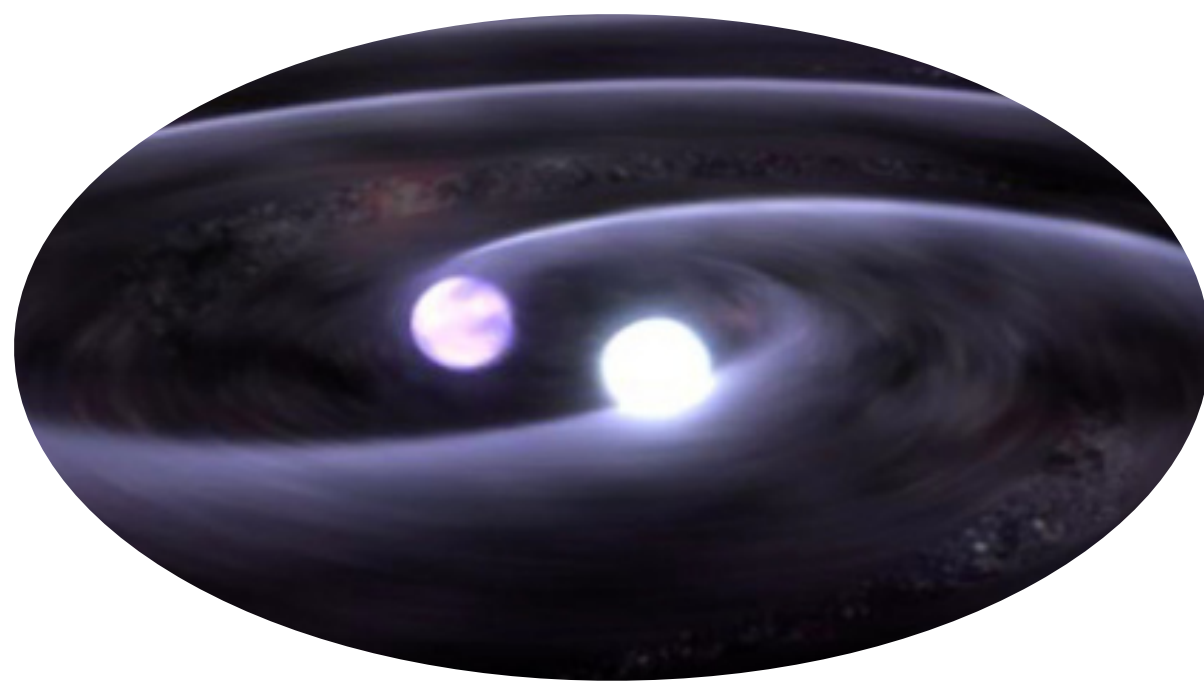
Supernovae Type Ia: Origin

PROGENITORS

CO white dwarf accreting mass.



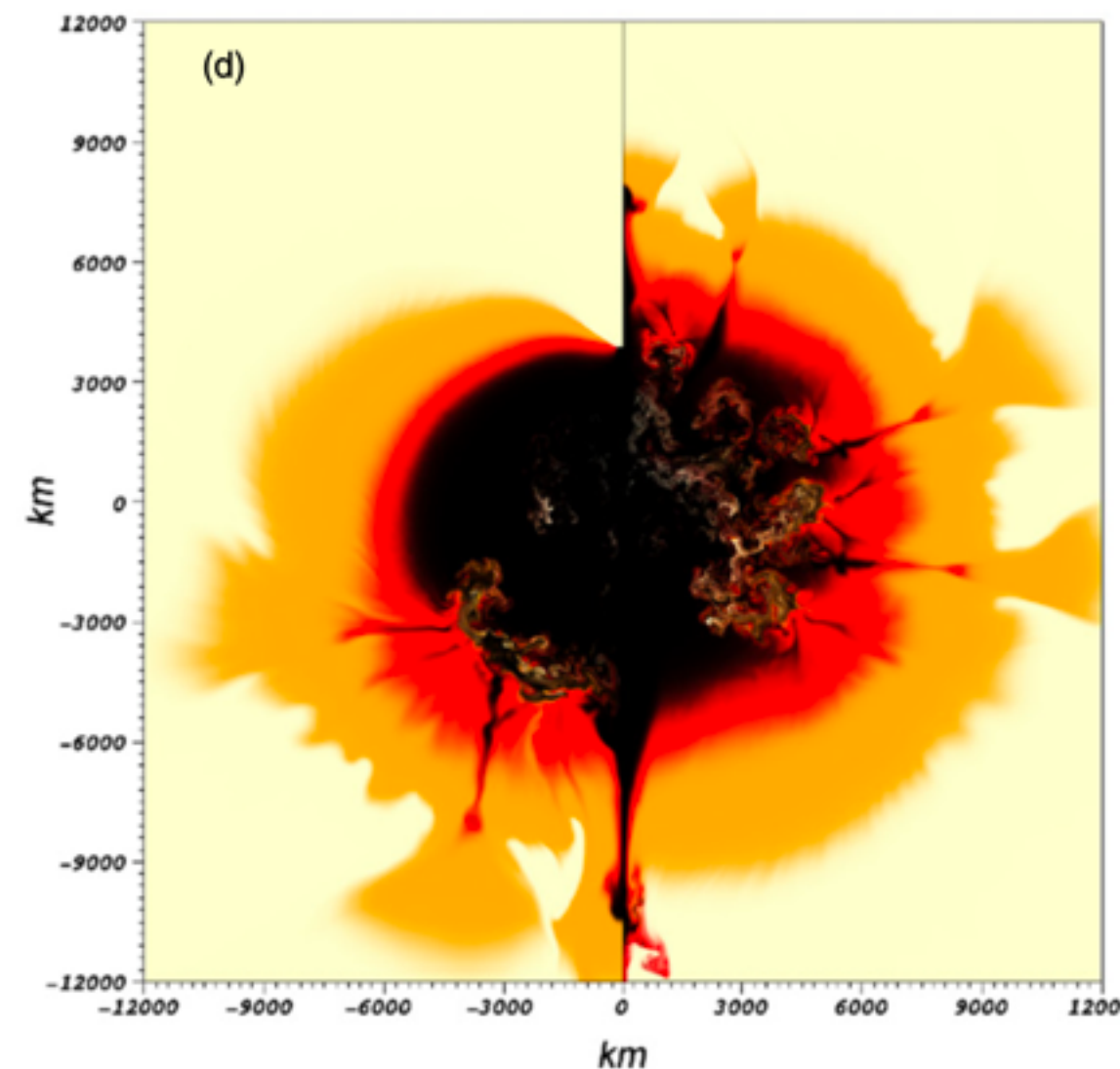
Single-degenerate
(e.g. Hosseinzaden et al, 2017)



Double-degenerate
(e.g. Roche & Garnavich, 2020)

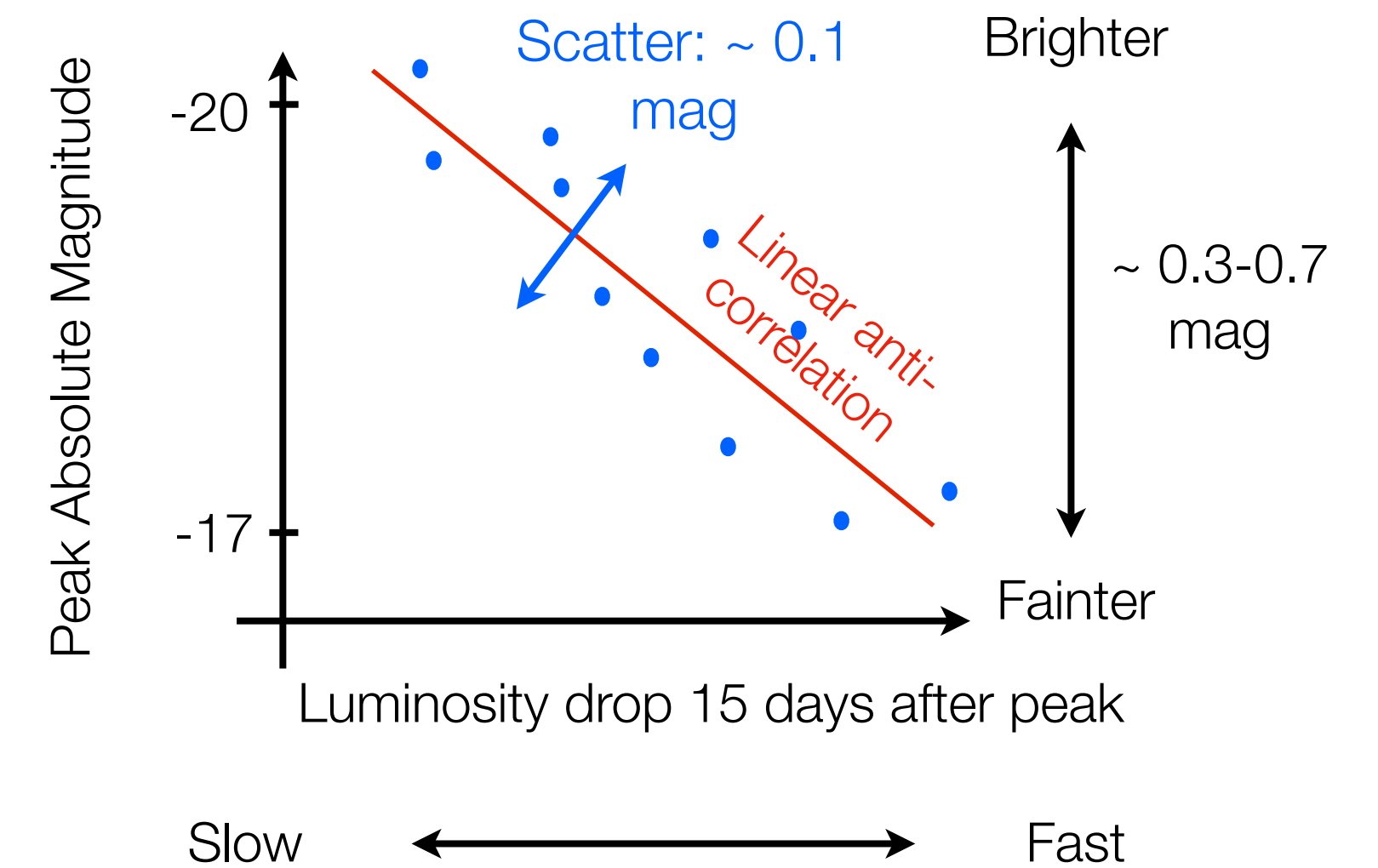
EXPLOSION

C detonation creates thermonuclear explosion.



Kruger et al (2012)

LUMINOSITY



Lightcurve powered by
radioactive decay of ^{56}Ni .

Higher core density $\Rightarrow$
Larger mass of ^{56}Ni & IGEs $\Rightarrow$
Higher luminosity & opacity $\Rightarrow$
SNIa brighter, slower to fade

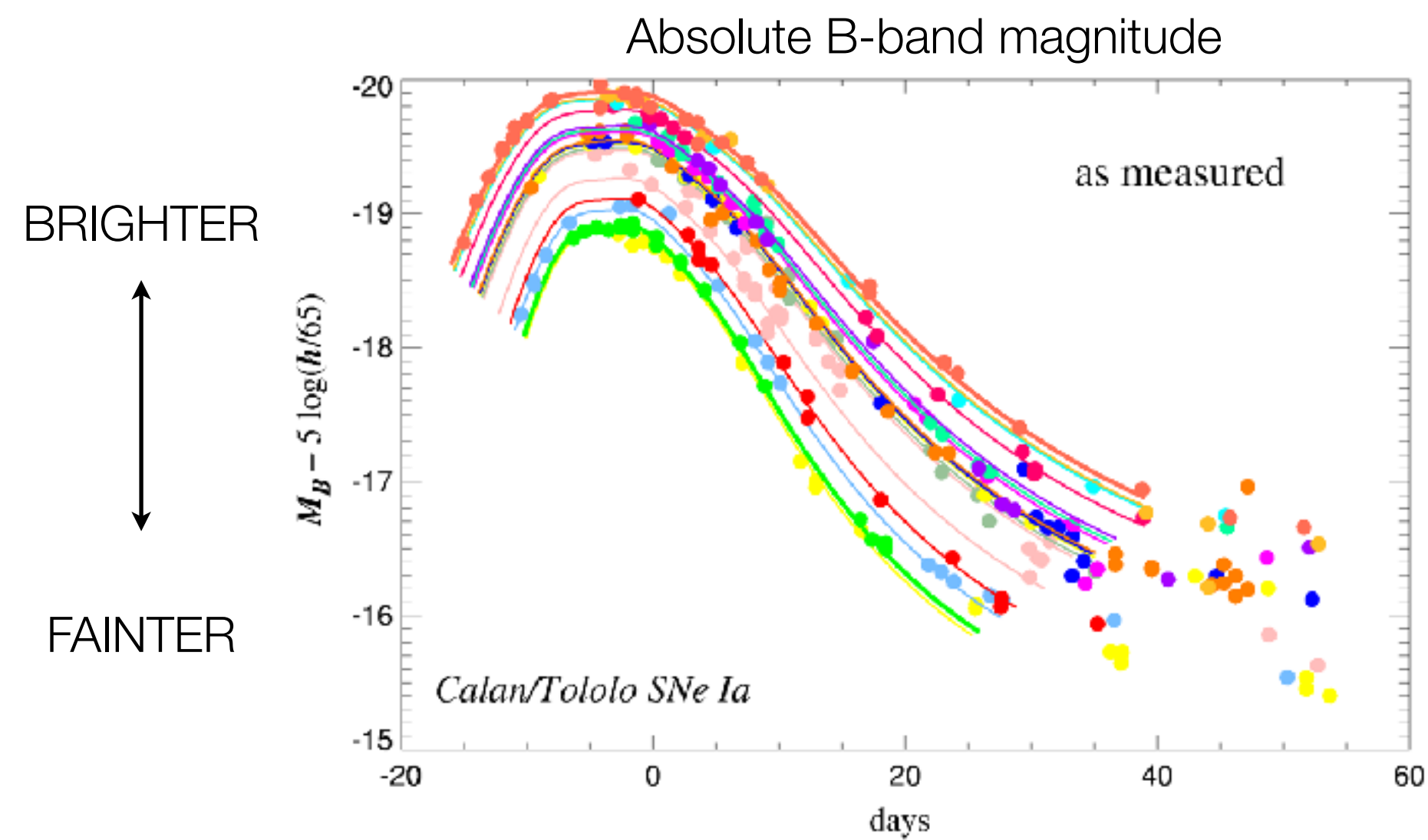
Phillips, ApJ 413 (1993) L105-L108

Standardization of SNIas

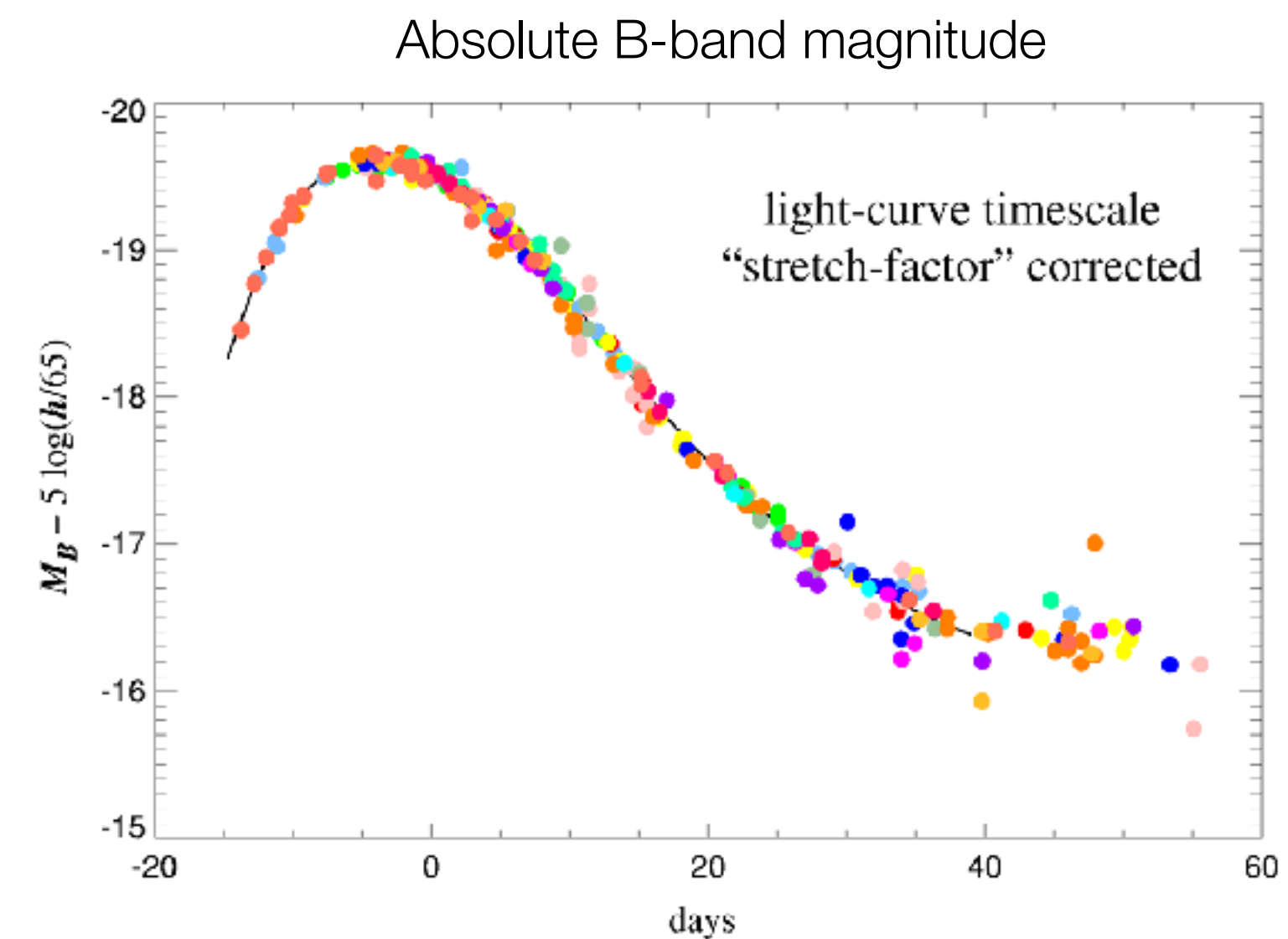
"Brighter SNIa are slow decliners"

Use the empirical 2D linear correlations between absolute magnitude and “stretch” and colour to standardise SNIas to within ~ 0.1 mag residual dispersion at peak

BEFORE CORRECTION



AFTER CORRECTION

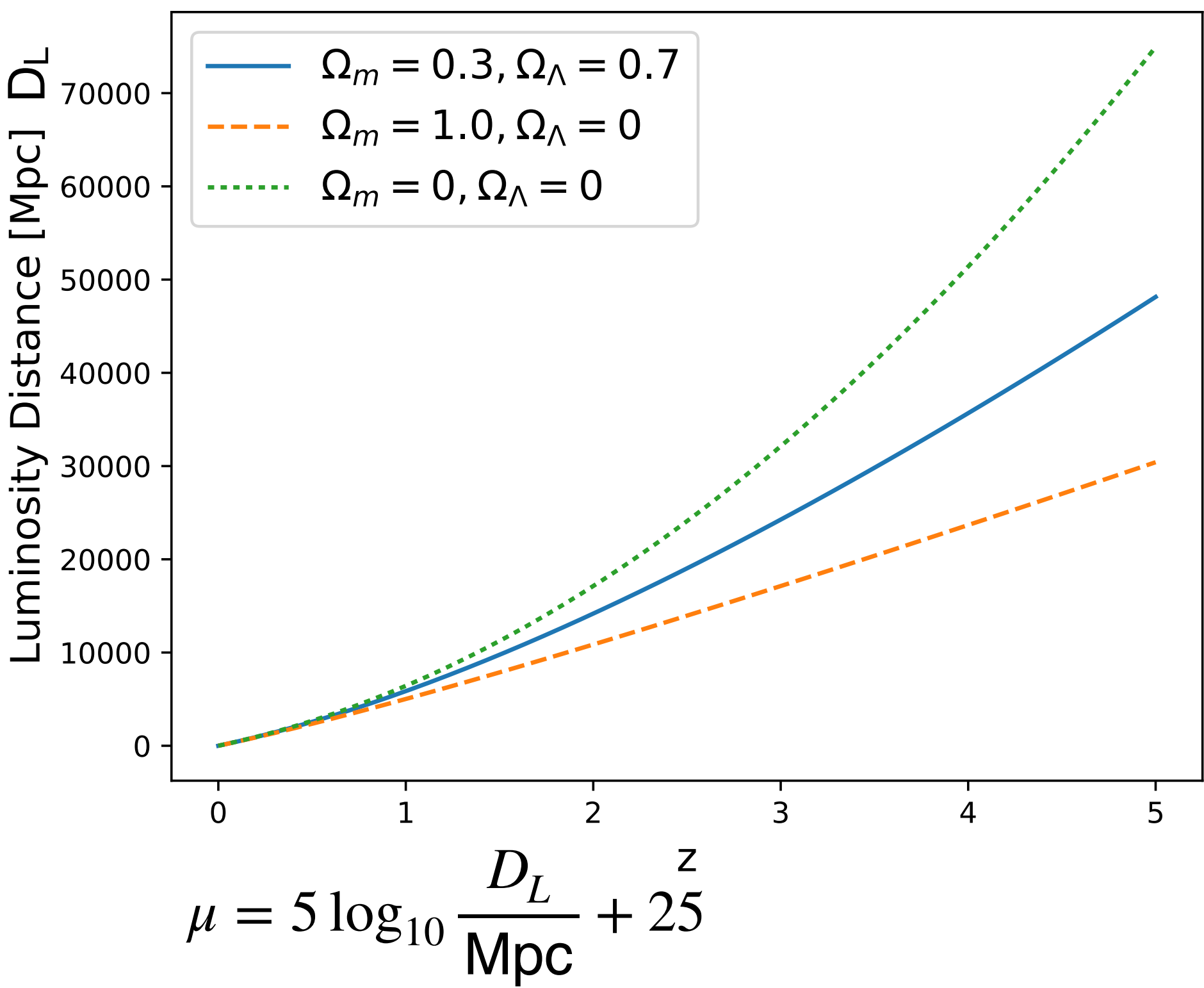


Kim et al (2007)

DISTANCE-REDSHIFT RELATION

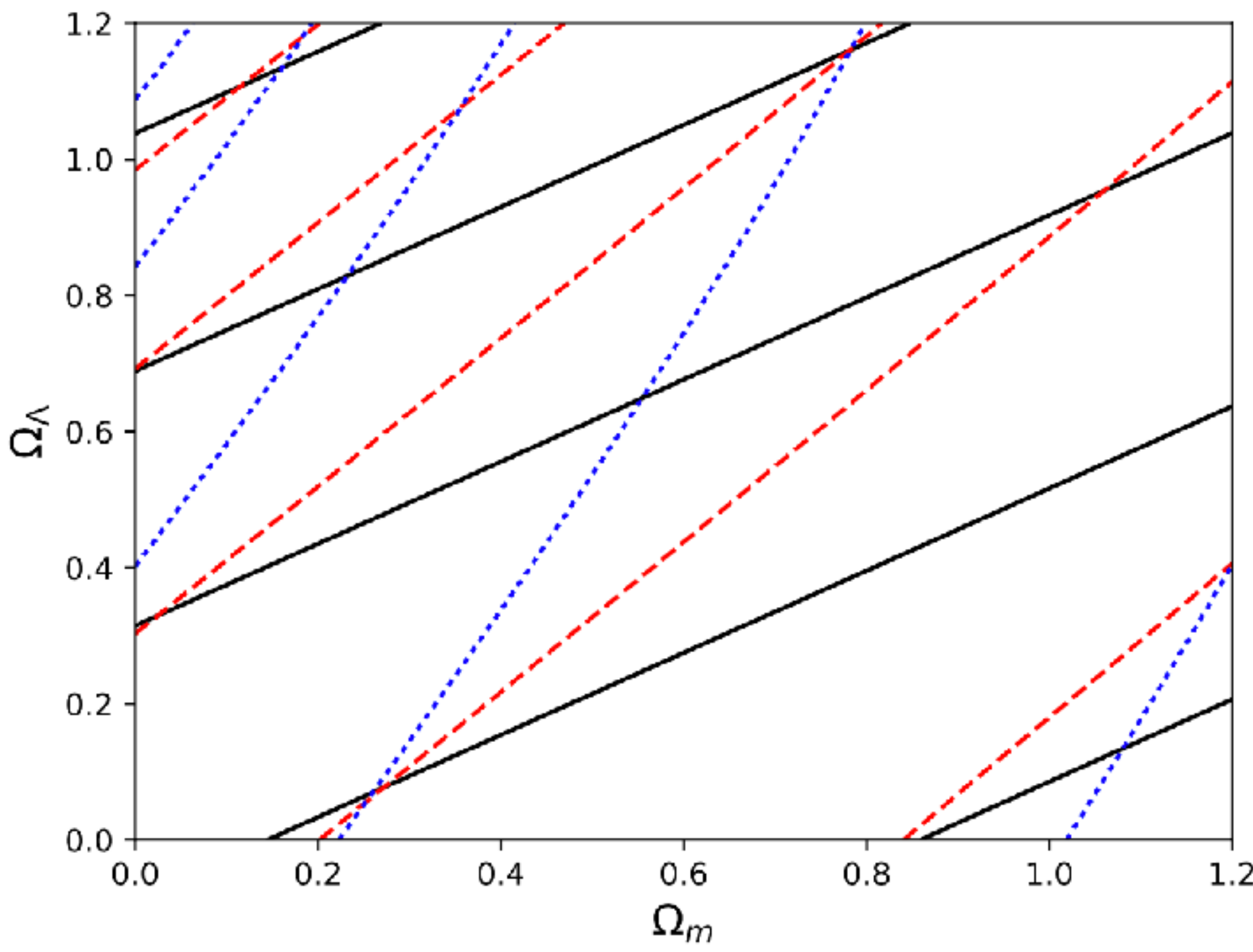
Measure redshift (z) and distance modulus, μ :

$$\mu = \underset{\substack{\text{Apparent} \\ \text{magnitude}}}{m_B} - \underset{\substack{\text{Absolute} \\ \text{magnitude}}}{M} + \underset{\substack{\text{Linear standardization}}}{\alpha x_1} - \beta c$$



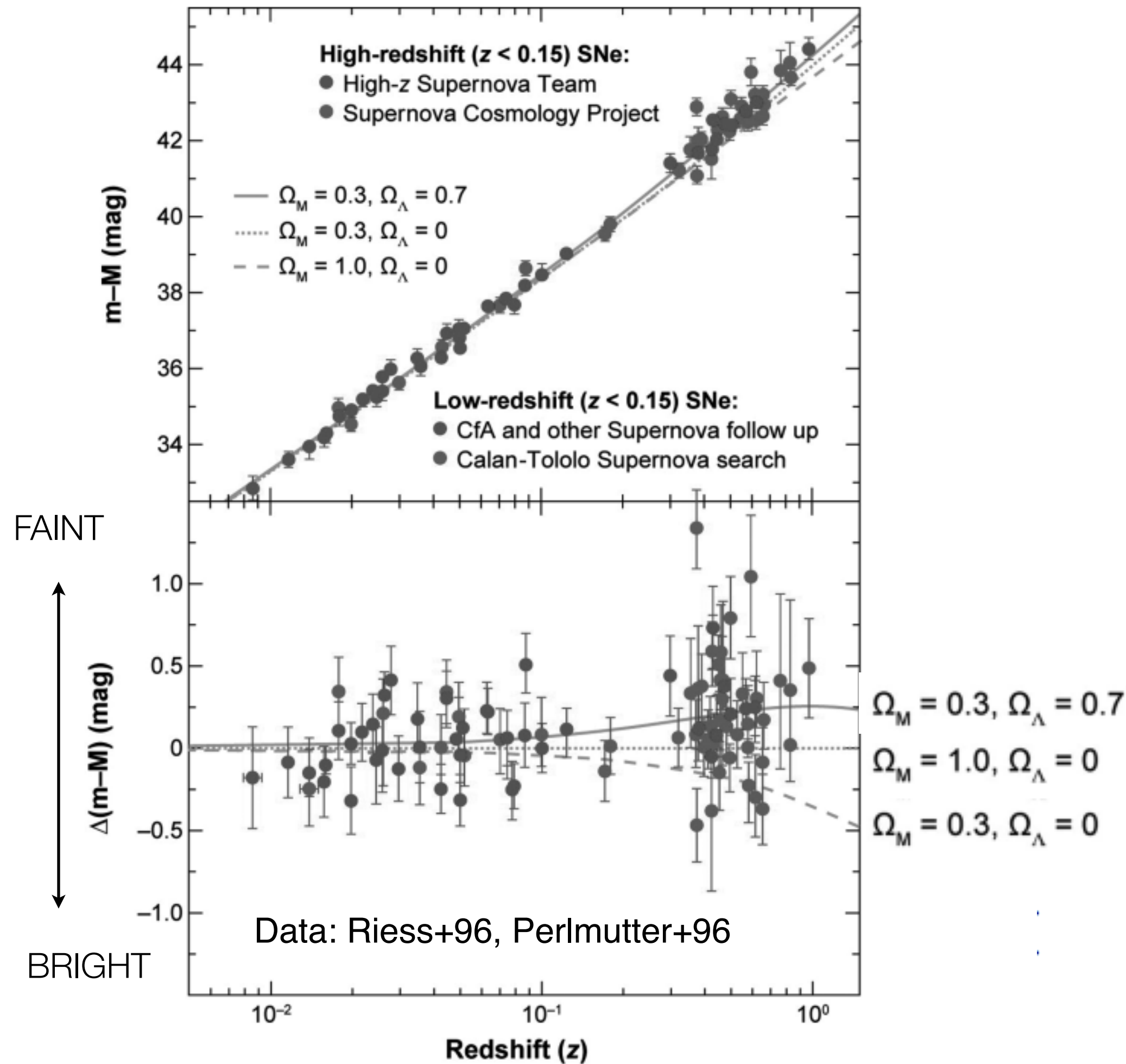
DIFFERENTIAL DISTANCE MEASUREMENT

$z = 0.1$ $z = 0.5$ $z = 1.0$

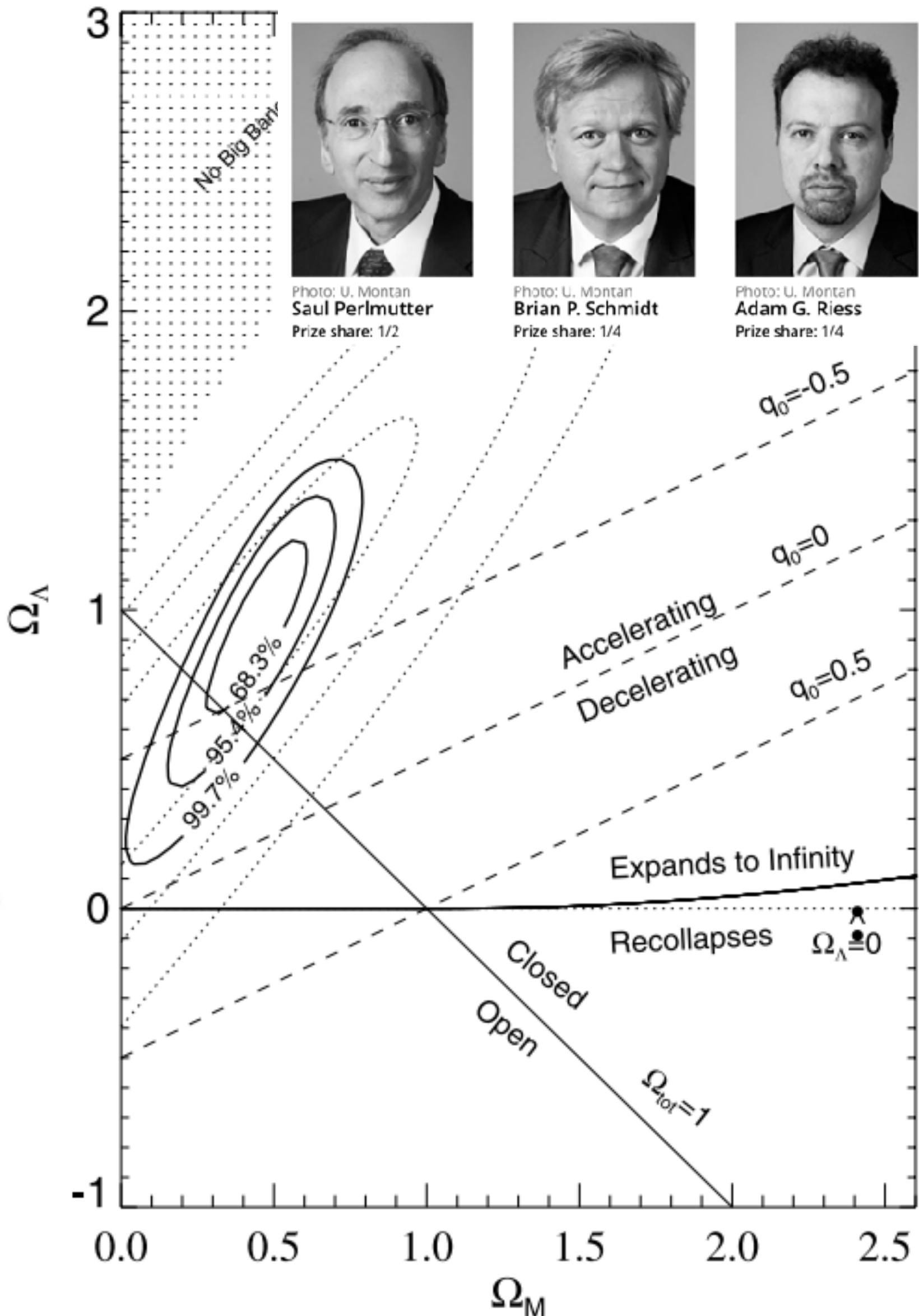


Contours of constant D_L at various redshifts

COLLECT THE NOBEL PRIZE



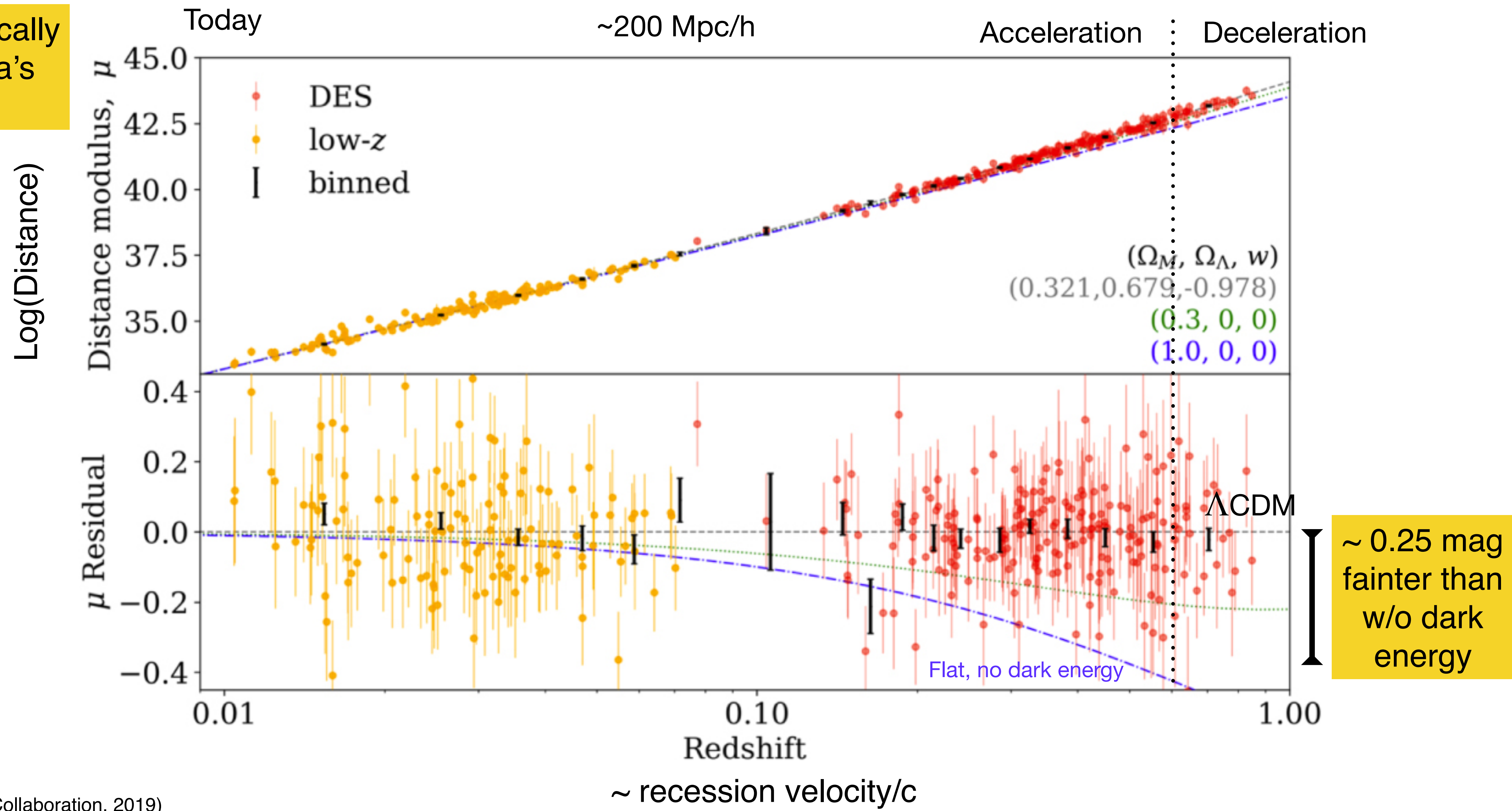
The Nobel Prize in Physics 2011



Riess et al, ApJ, 607:665-687 (2004)

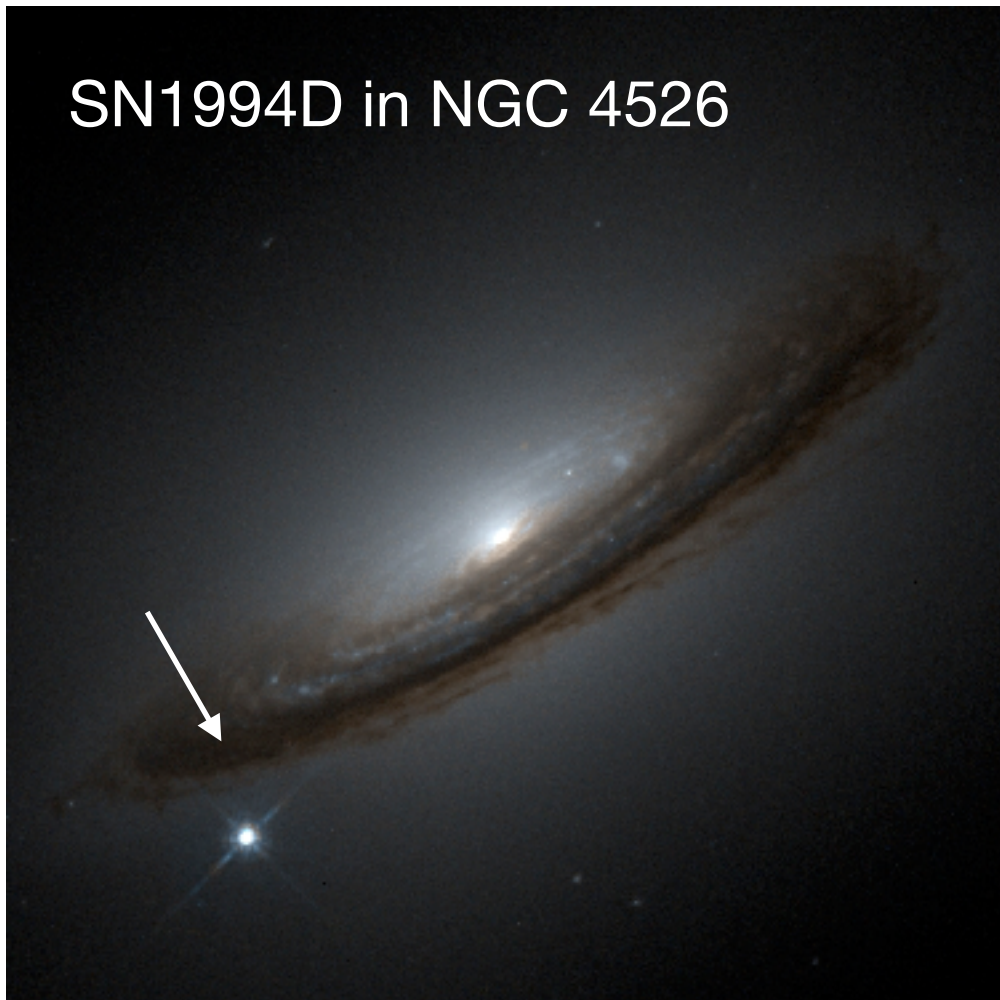
Distance-Redshift Relation Measurement

Spectroscopically confirmed Ia's only



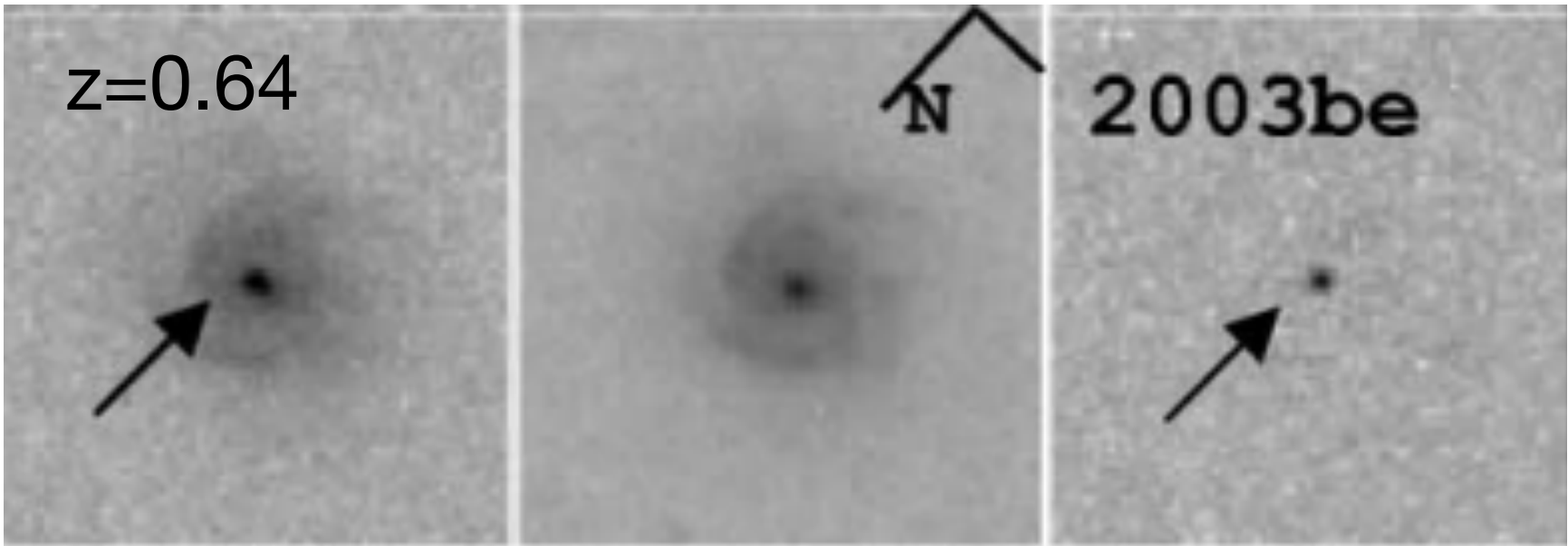
Supernovae Type Ia: Observations

DETECTION



Treffers et al (1994); imaged by HST

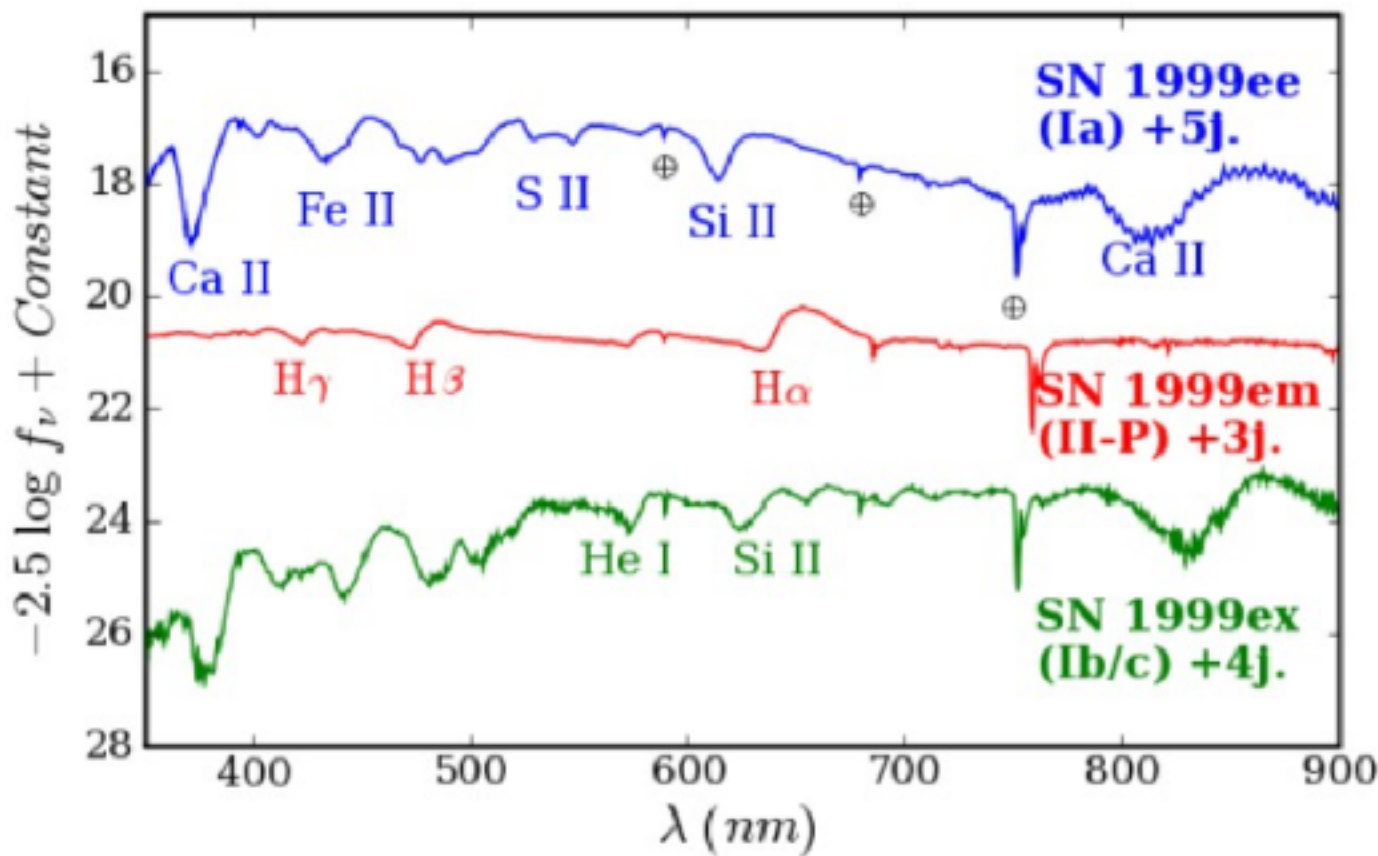
During After Subtracted



Riess et al, ApJ, 607:665-687 (2004)

TYPING

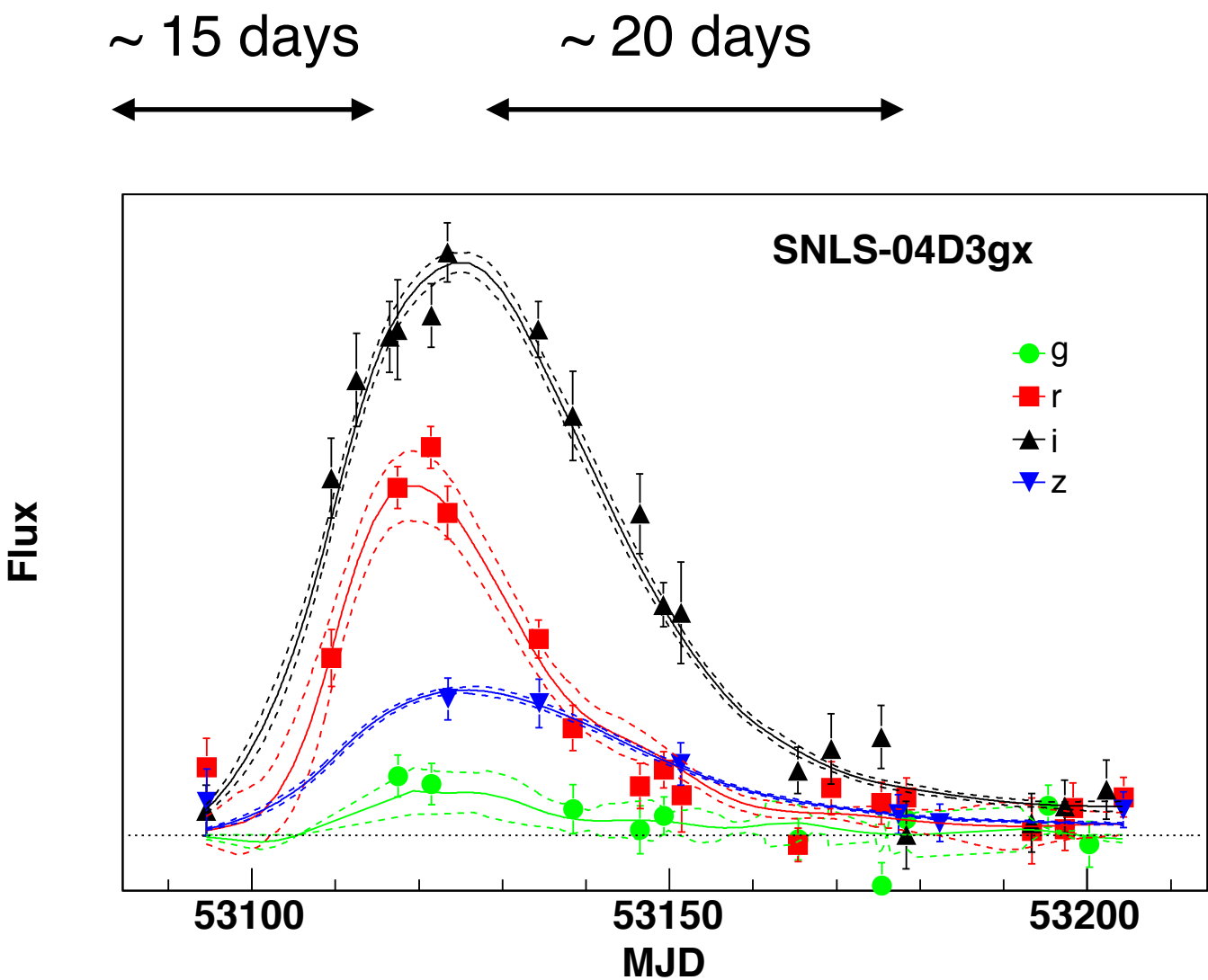
Spectra: Type Ia have no H lines



Credit: Julien Guy

Confirming Ia is easy with spectra.
Much harder with just photometry (i.e., low-res spectral information): **only probabilistic classification**

FOLLOW-UP

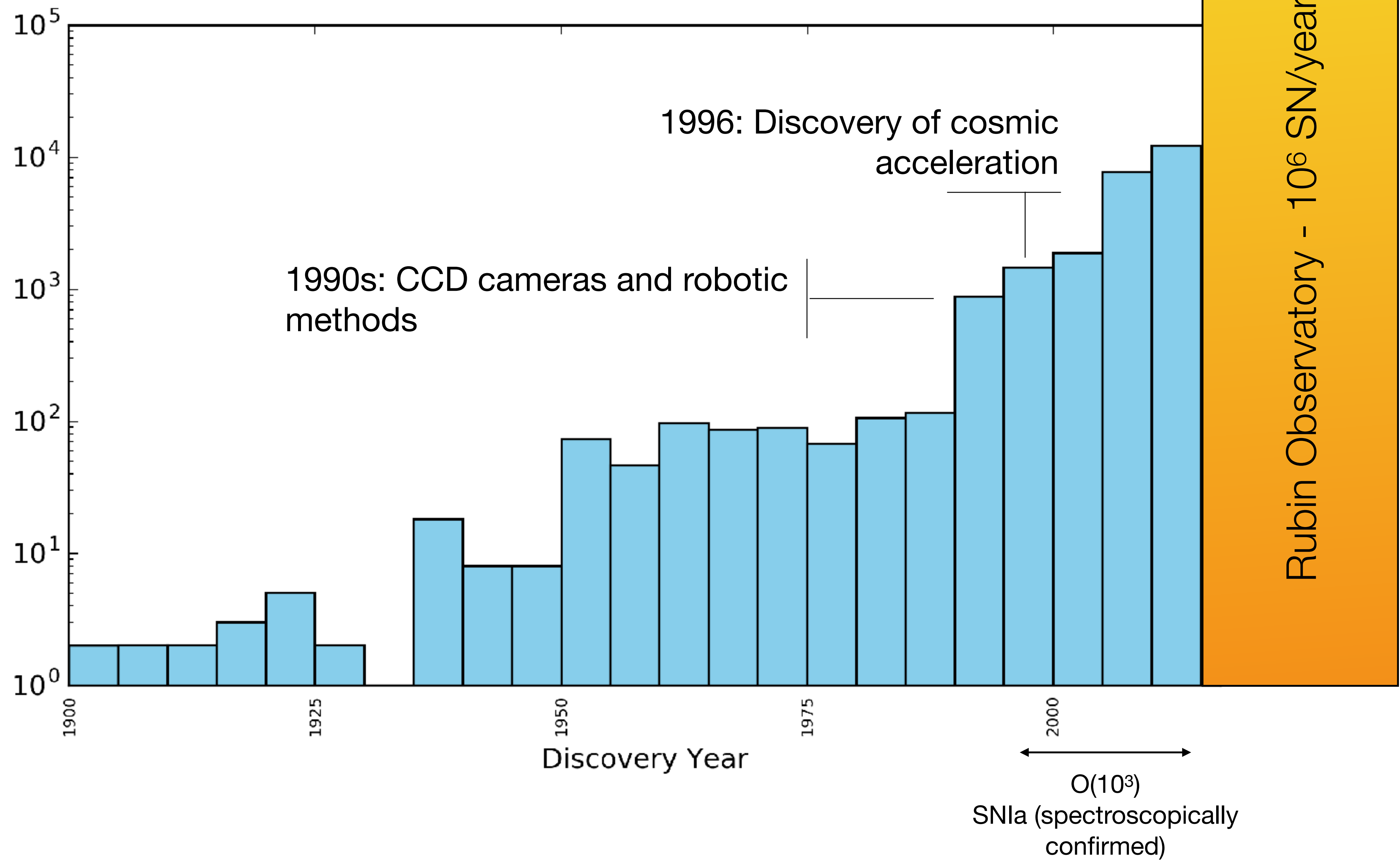


Guy et al (2007)

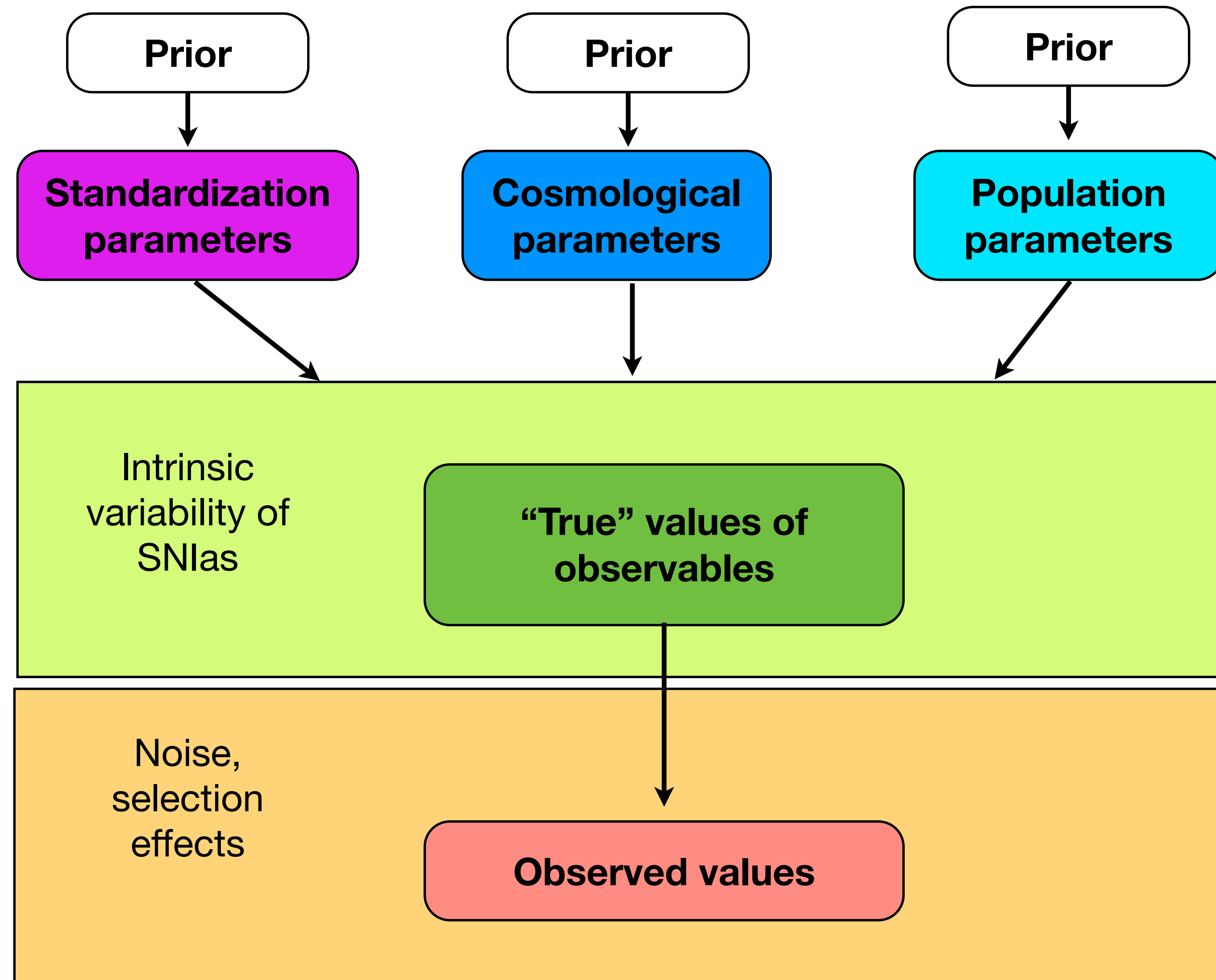
Lightcurves: time-evolution of brightness in several colour filters

Used for standardization and cosmological inference

Supernovae Discoveries Over Time



Bayesian Hierarchical Modelling of SNIa data



SNIa population distributions
Environmental properties (age, metallicity, SFR, host type)

Latent variables:
unknown and unobserved, are integrated out during the inference

Noisy data subject to truncation

For more details, see:

BHM: March, RT et al (2011)

Unity: Rubin et al (2015)

BAHAMAS: Shariff, RT et al (2016); Rahman, RT et al (2022)

Simple-BayeSN: Mandel et al (2017)

Steve: Hinton et al (2019)

Upcoming:

MALFOI: Karchev, RT & Weniger (soon)

The Problem:

We want to classify Type Ia vs non-Ia **reliably** and **efficiently** from light-curve data alone.

BUT:

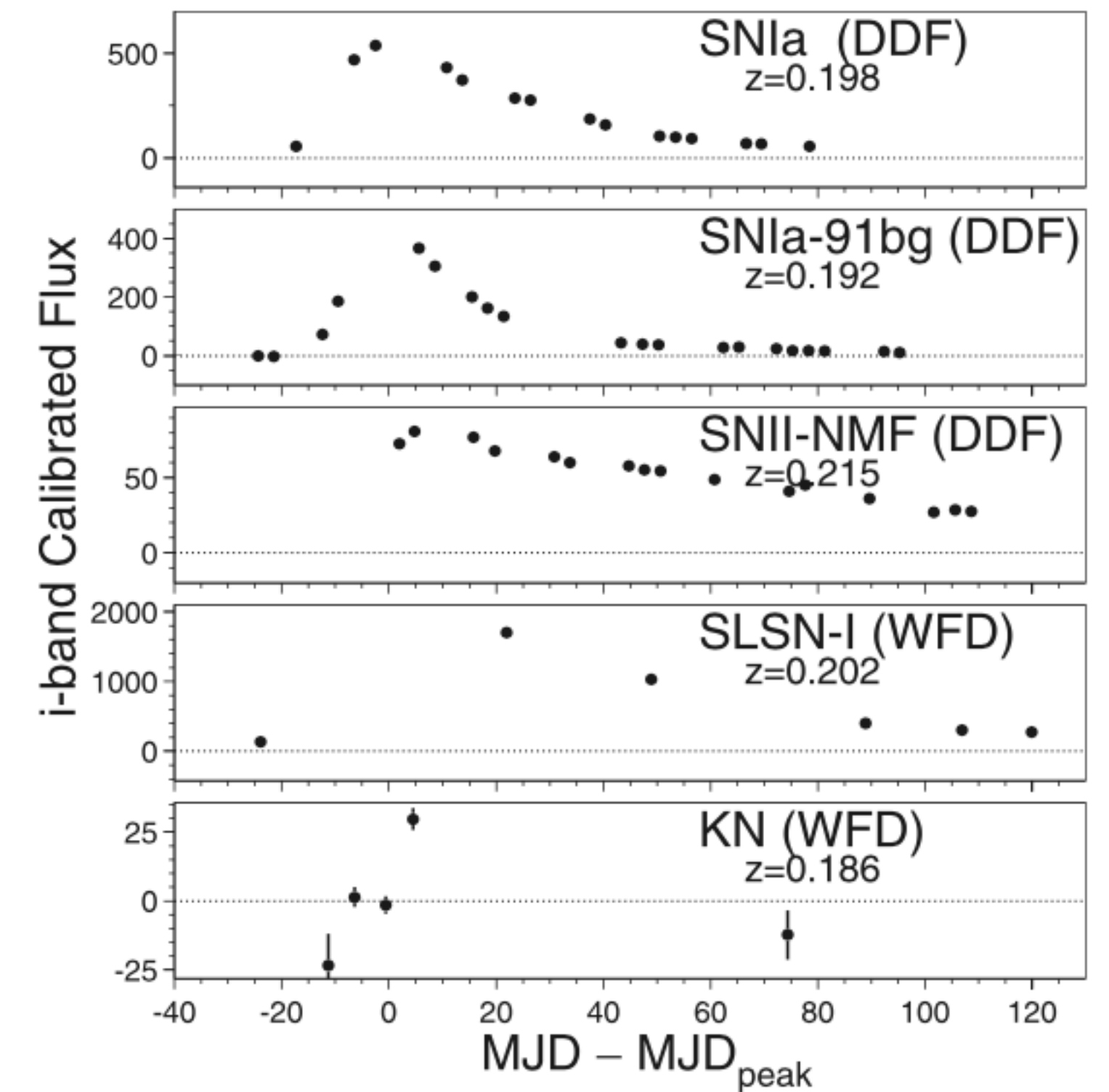
Spectroscopic training set is non-representative.

Classification challenges:

The Photometric LSST Astronomical Time-series Classification Challenge PLAsTiCC (Kessler et al, 2019)

Supernova Photometric Classification Challenge (Kessler et al, 2010)

Simulated light-curves

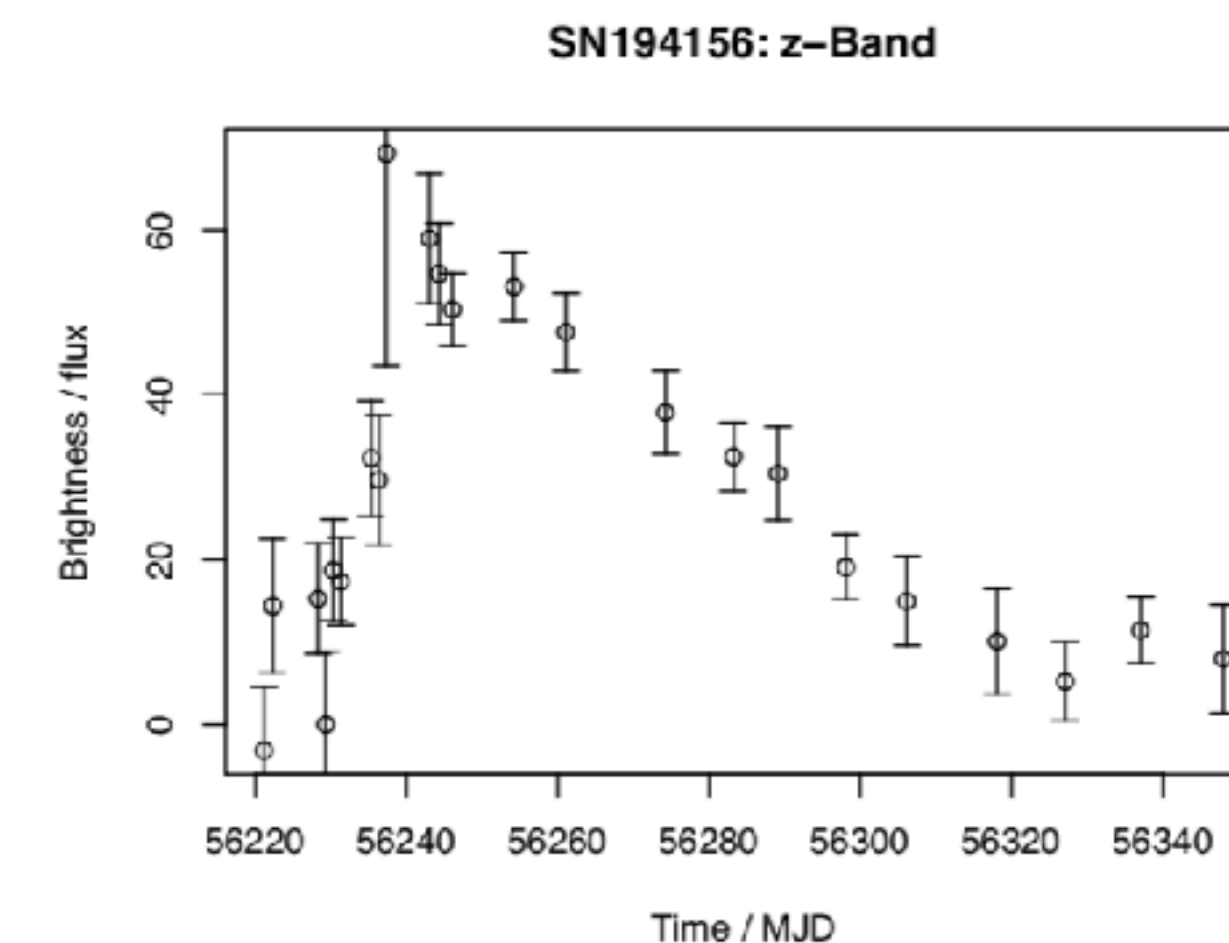
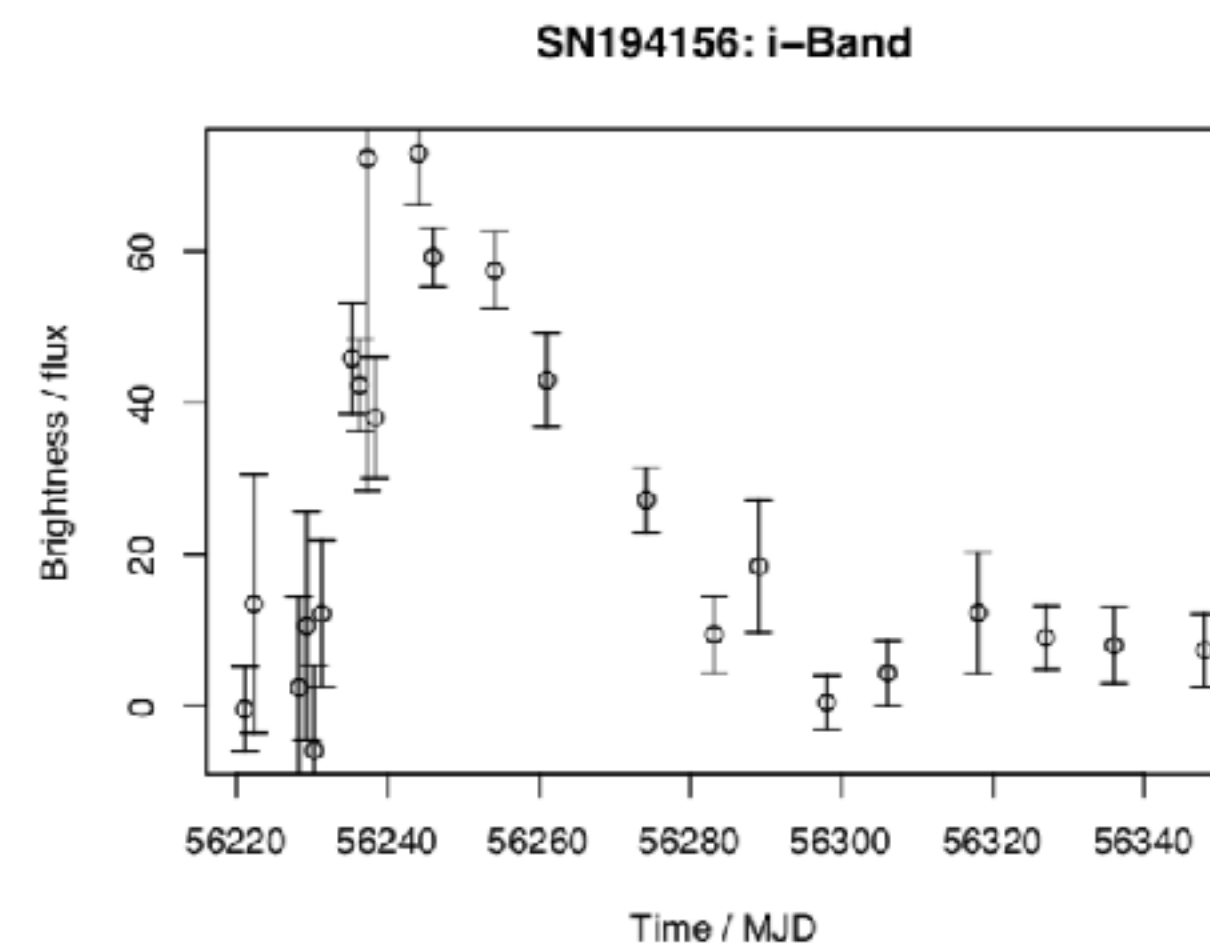
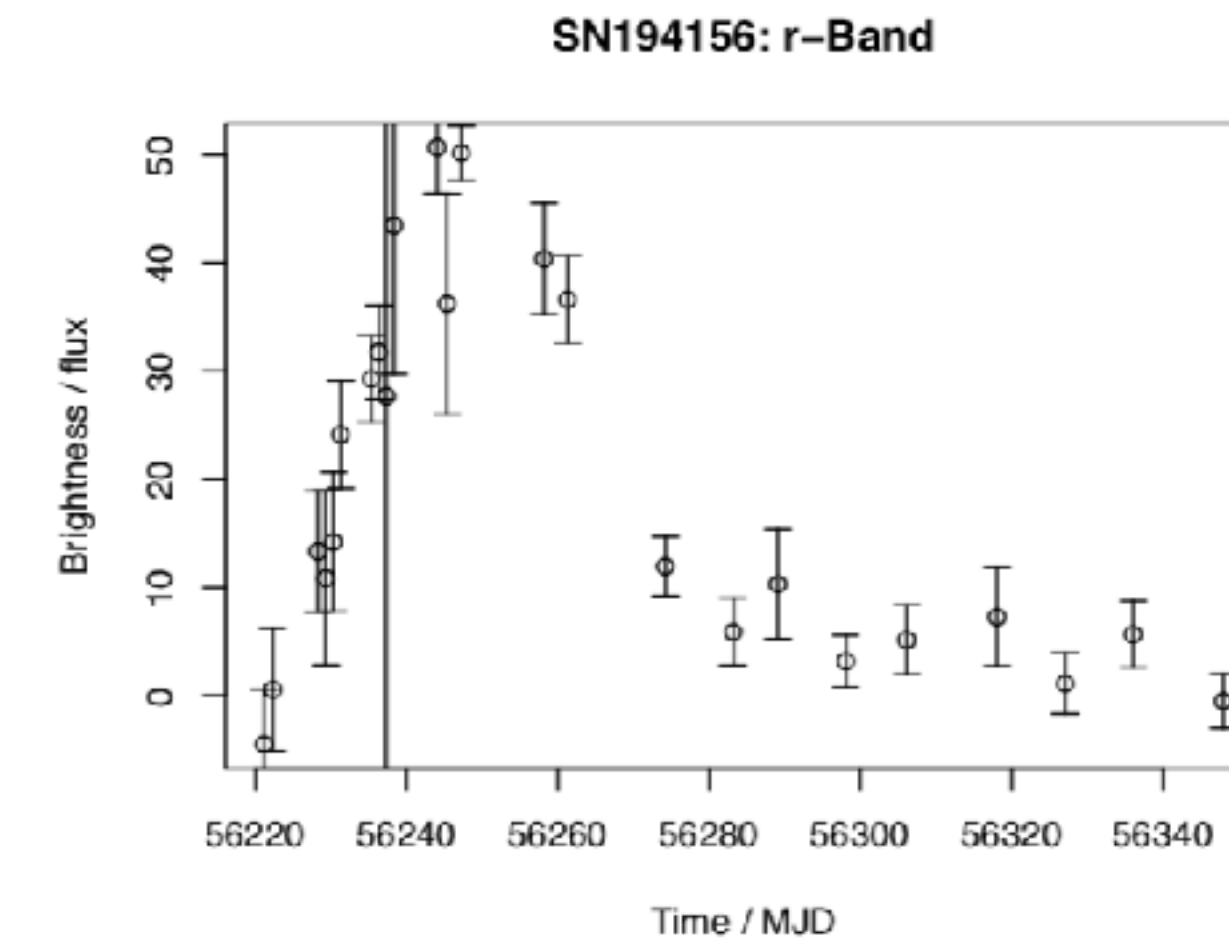
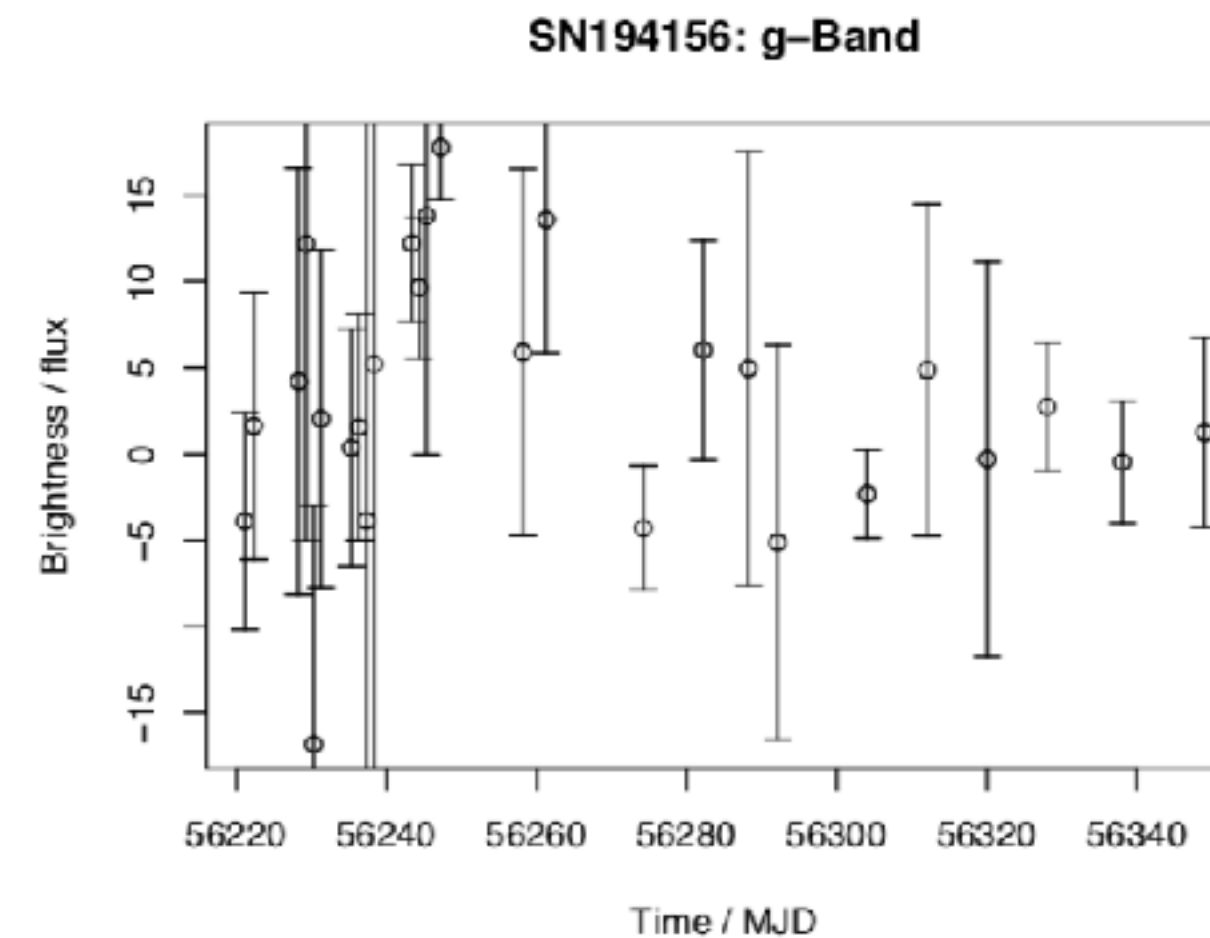


Kessler et al (2019)

The Future: Photometric SNIa Cosmology

Example of simulated LC data

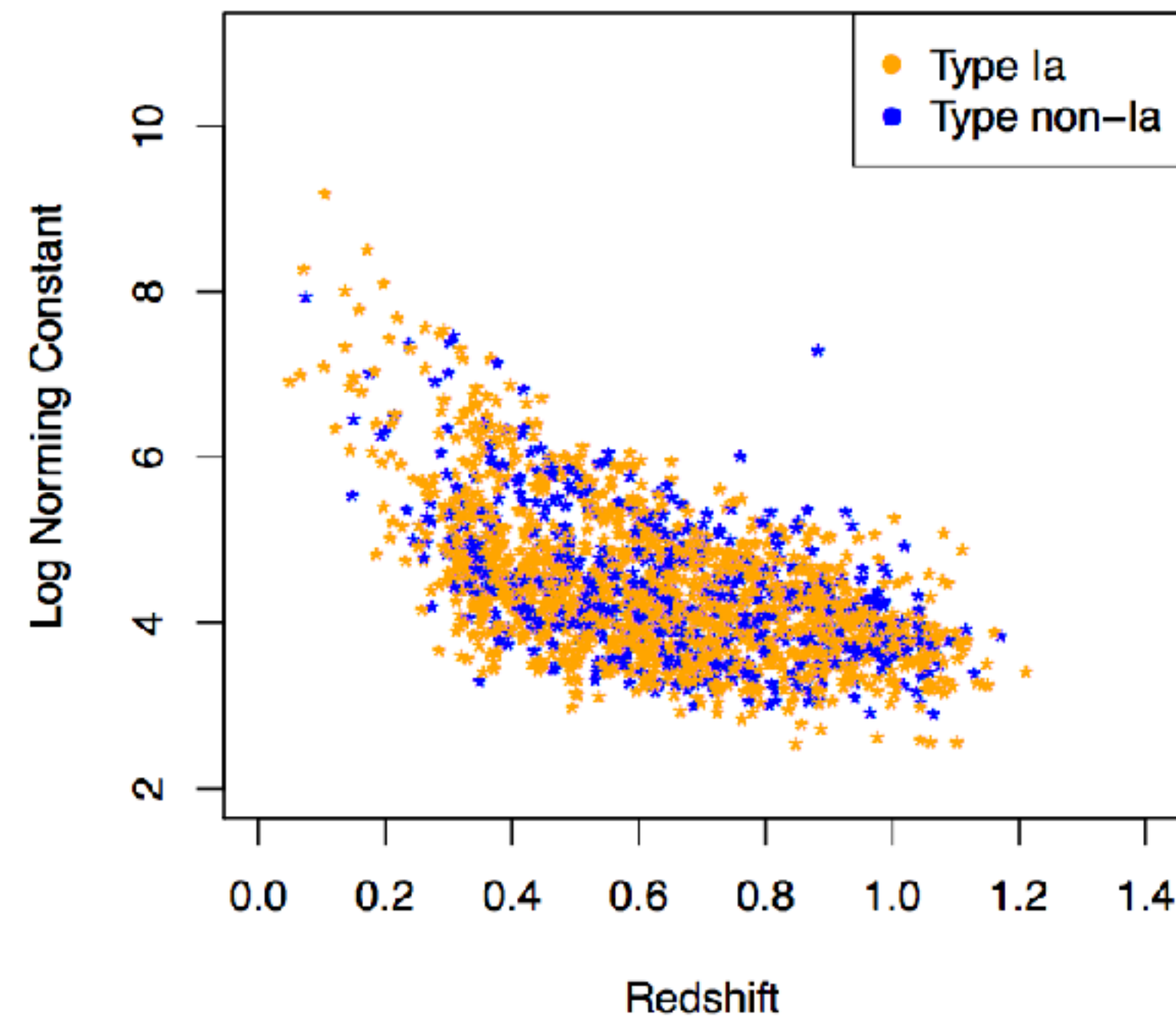
- SNIa identification relies on observationally expensive spectroscopy
- In the future, we won't have spectra for all SNIa candidates (DES: 3,000 SNIa over 5 yrs; LSST: 10,000 SNIa/yr)
- SNIa classification needed from multi-band imaging alone
 - “SN Classification Challenge” (Kessler+10)
 - more recently: LSST Kaggle comp (2018)



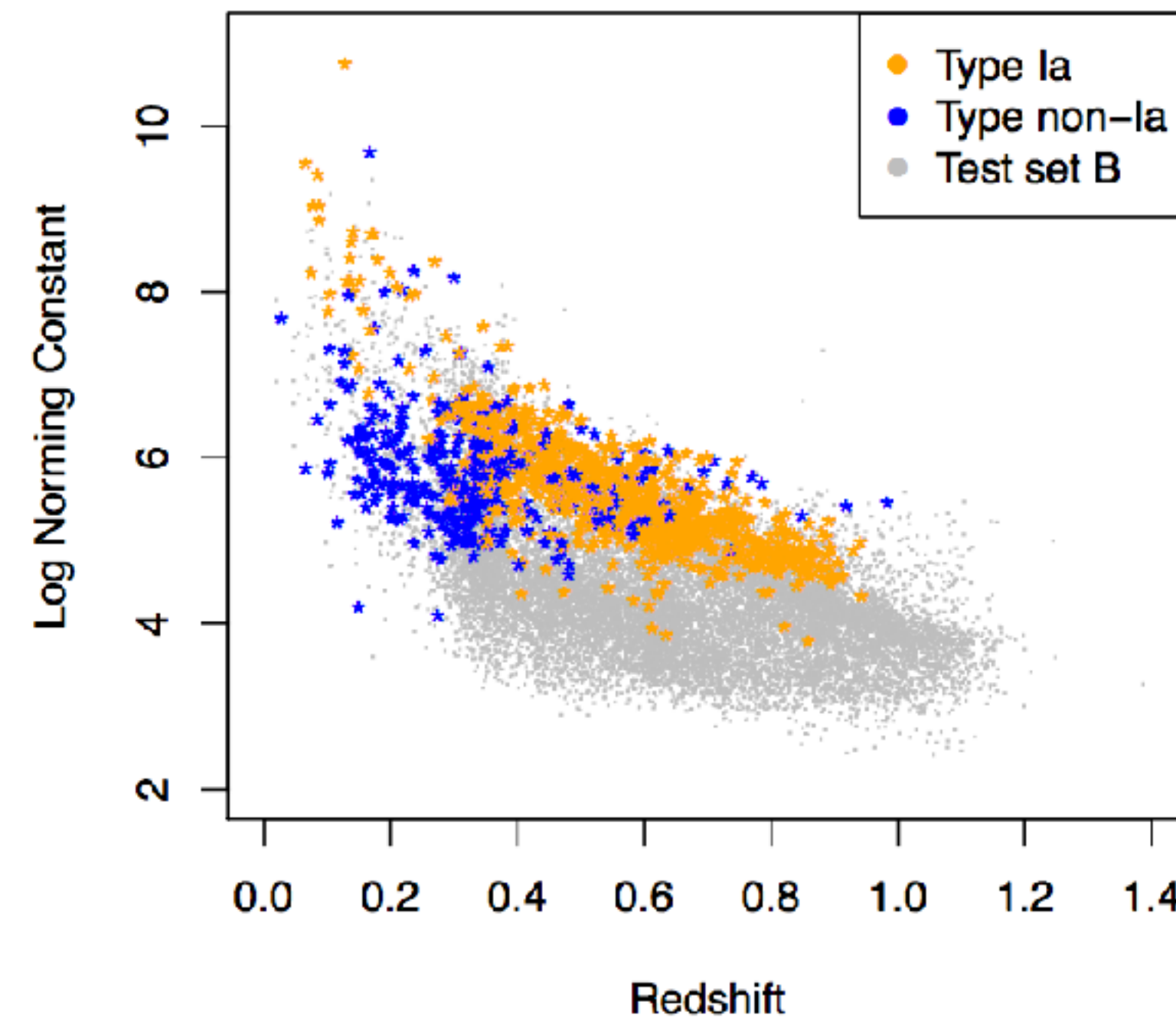
The Future: Photometric SNIa Cosmology

- **Problem:** Training set is biased. Especially at high z , more SNIa's than in the population, hence non representative

Random training data
(unavailable)

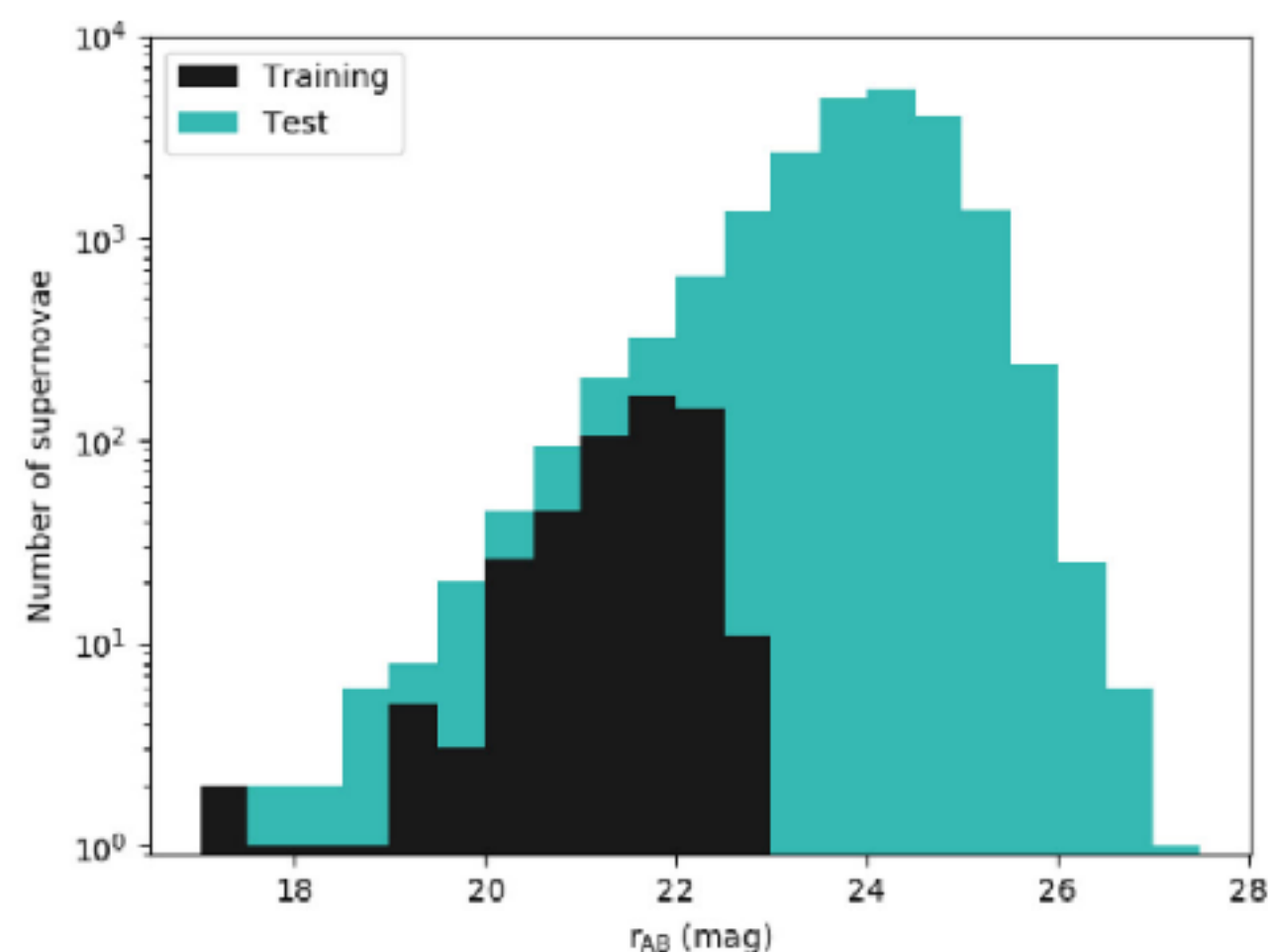


Biased training data (colour)
vs Test Set (gray)

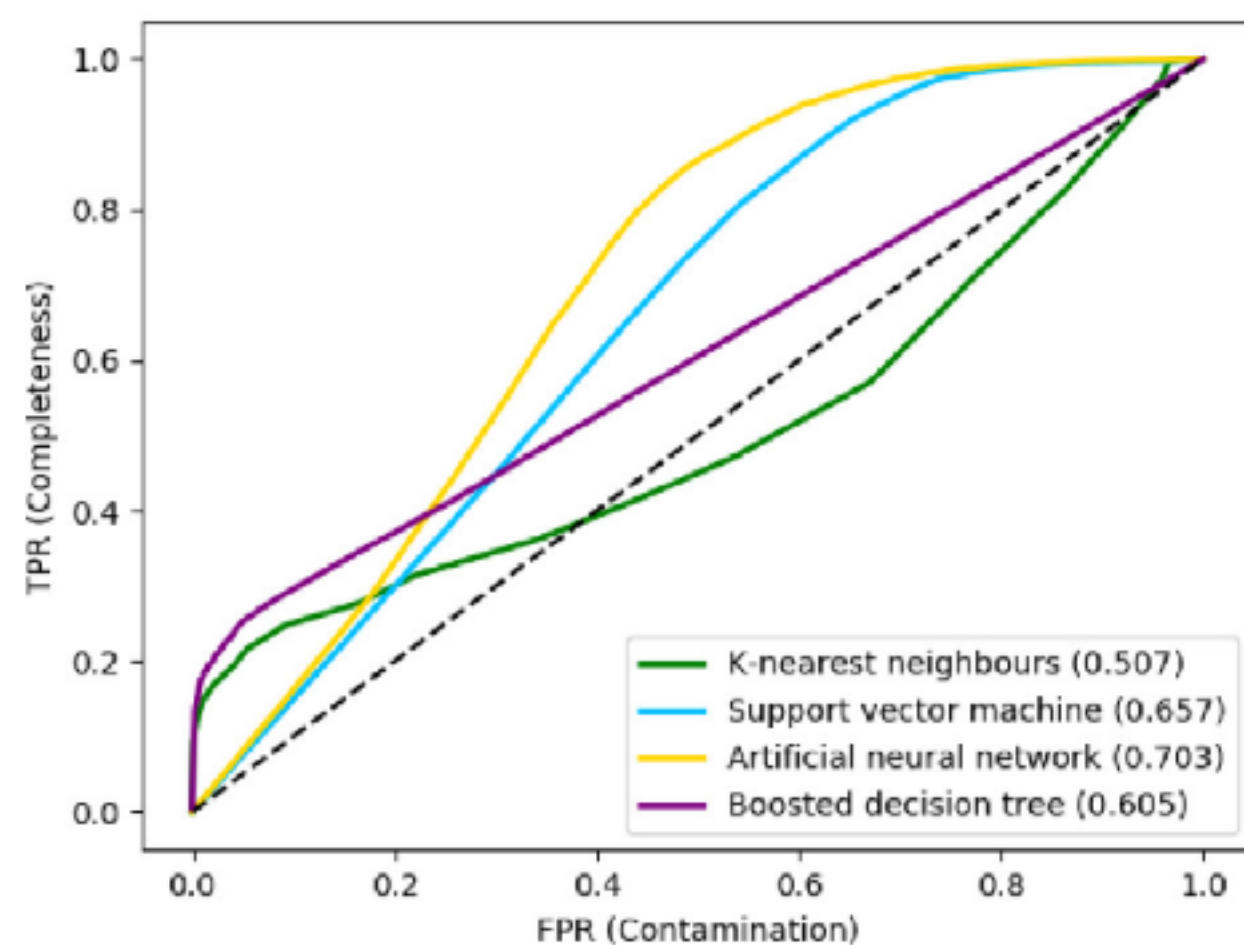


Solution 1: data augmentation

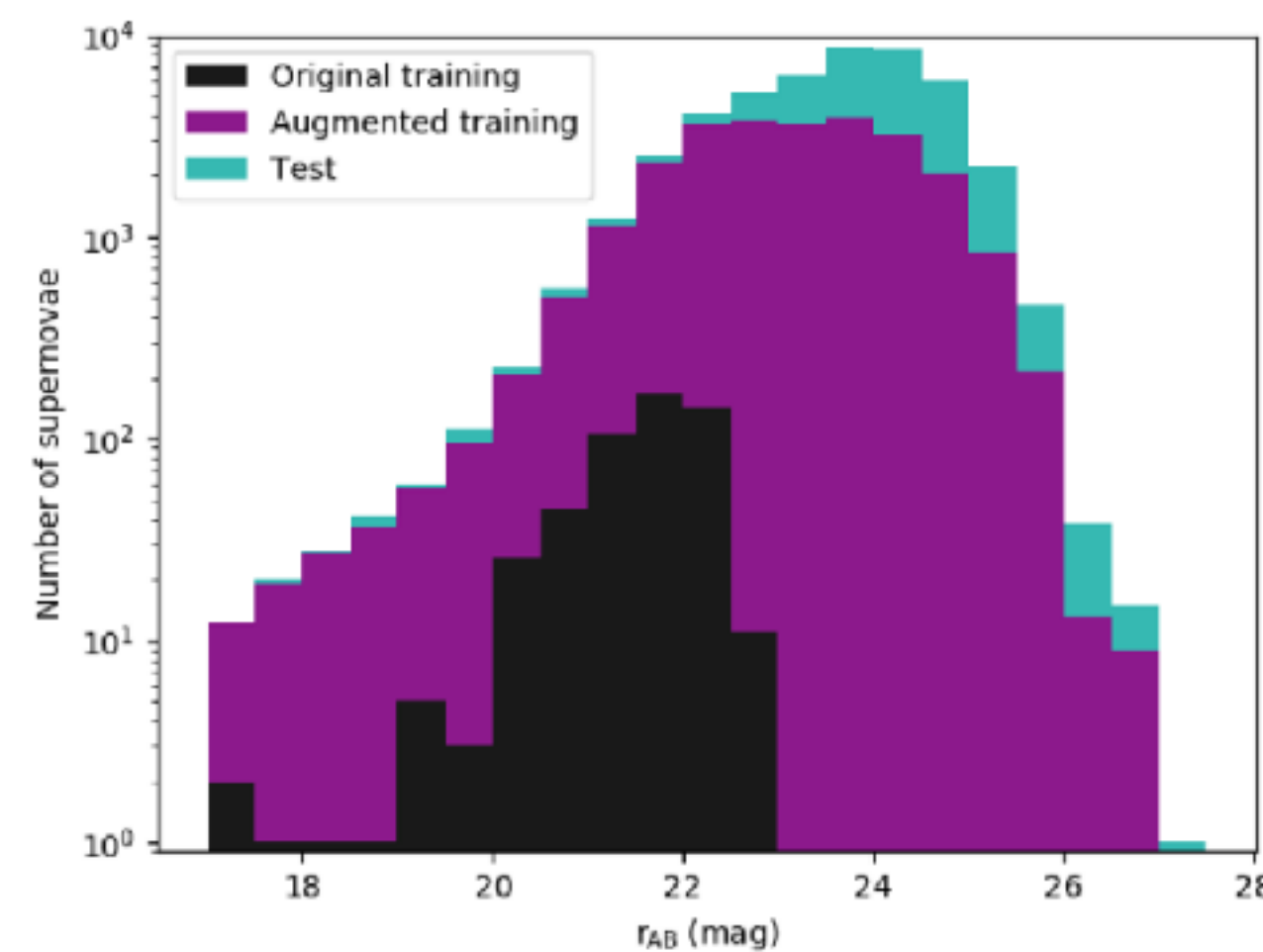
Unrepresentative spectroscopic training set
leads to poor classification performance



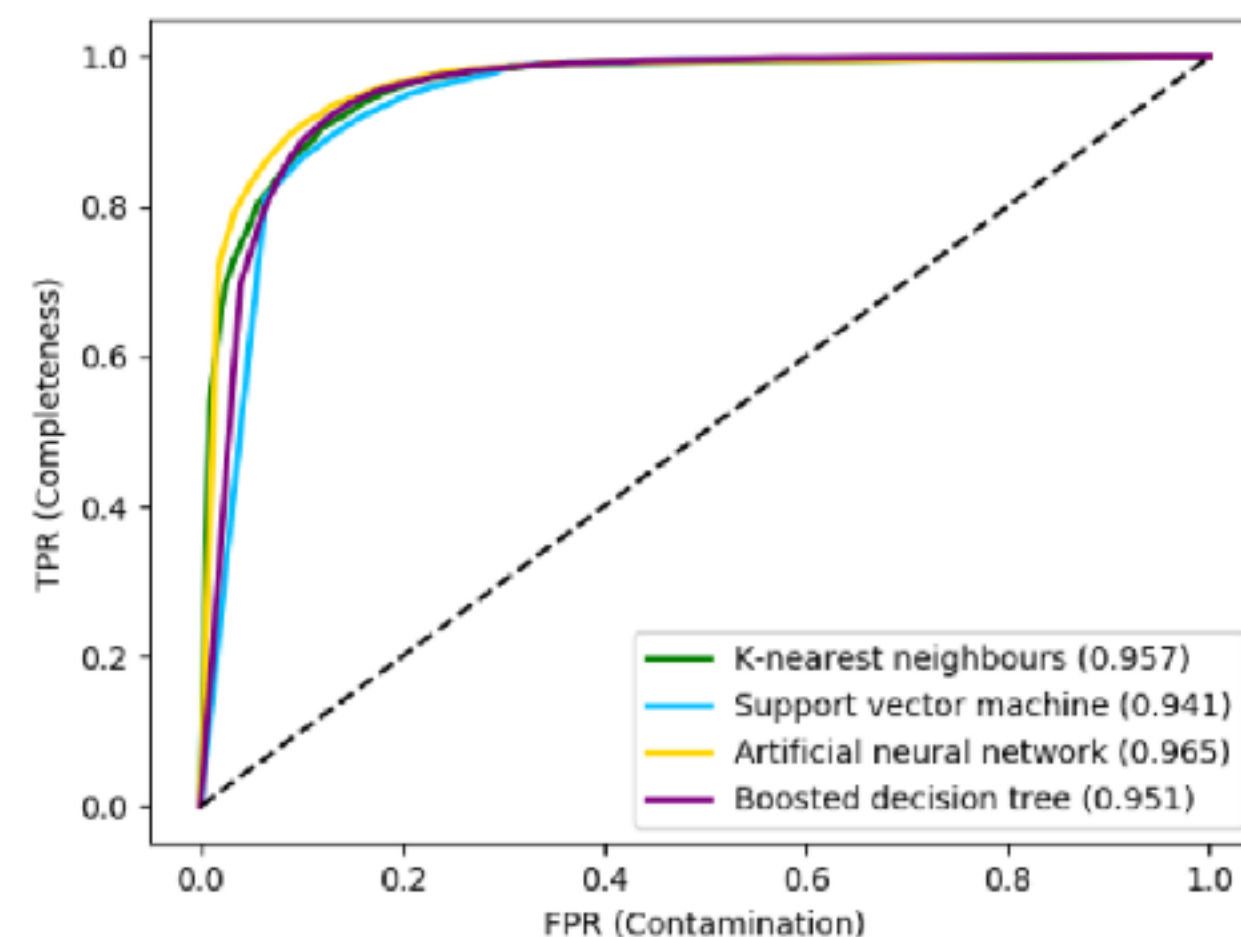
(a)



+ 50-fold data augmentation improves
massively the AUC



(a)



However:

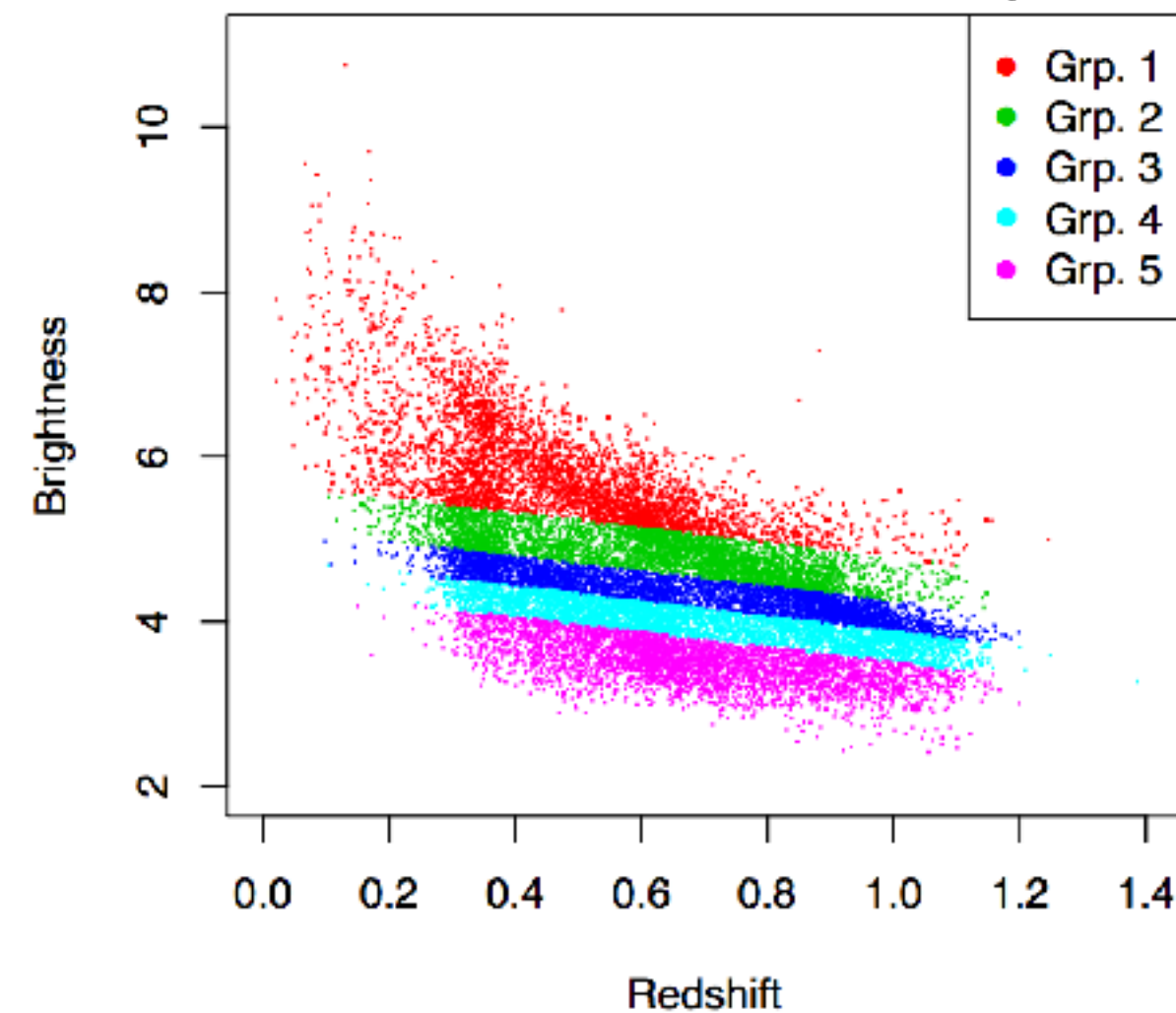
Successful
augmentation relies
on a good light-curve
model (e.g. GPs)
AND the assumption
of no astrophysical
evolution with z

A first solution: STACCATO

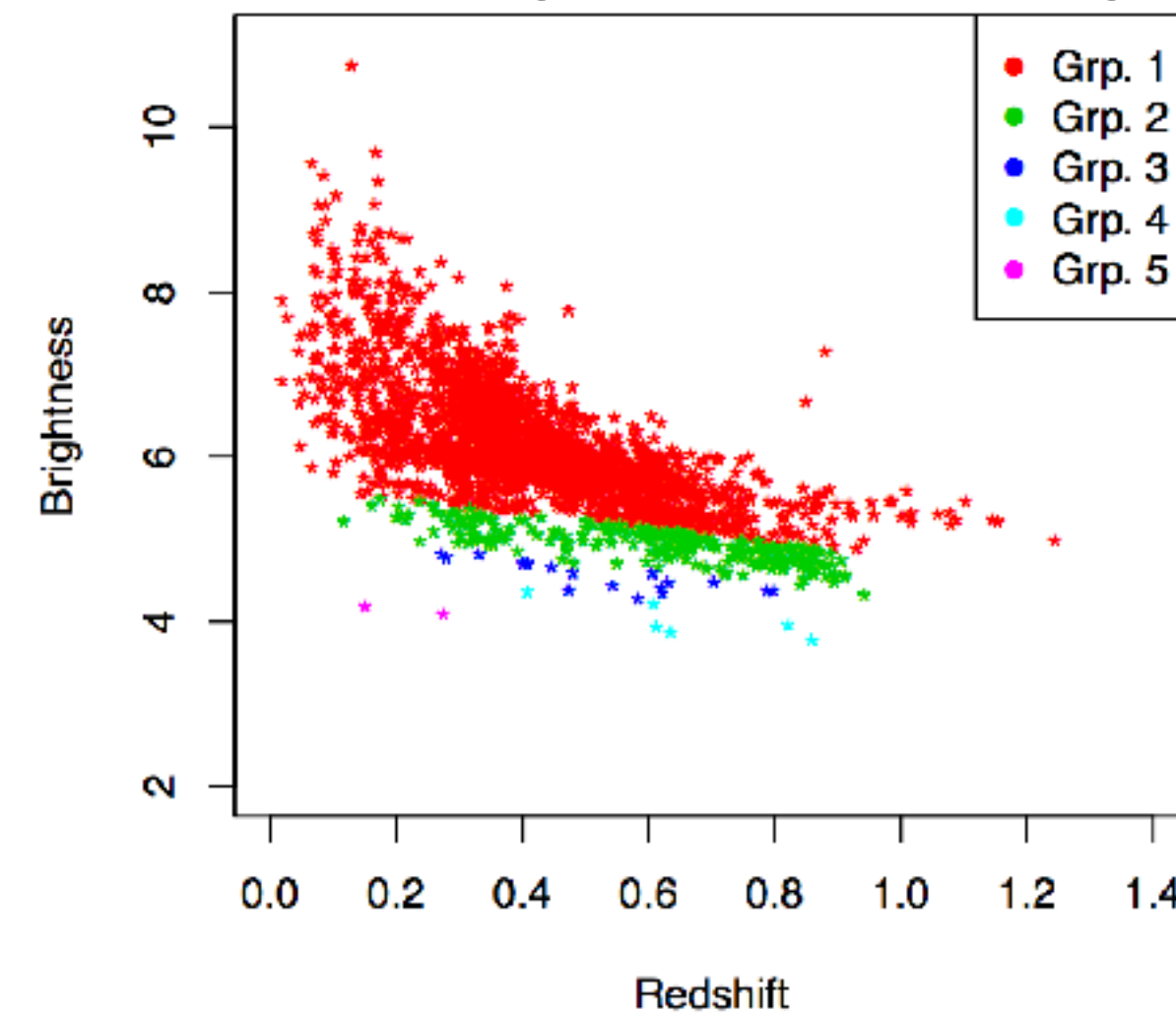
- Our first solution (Revsbech, RT, van Dyk, MNRAS, 473, 3, 3969 (2018), arxiv:1706.03811): **SynThetically Augmented Light Curve ClassificATIOn** (STACCATO) approach
 - Fit light curve with Gaussian Process (GP)
 - Compute Diffusion Map (to quantify similarities between LCs), Richards+12
 - Perform Random Forest Classification on diffusion coordinates
 - **New:** Group SNe according to **Propensity Score** (probability of belonging to the training set): bias within groups reduced by 90% (Rosenbaum & Rubin, 84)

Propensity Score
Grouping:

Test data partitioning

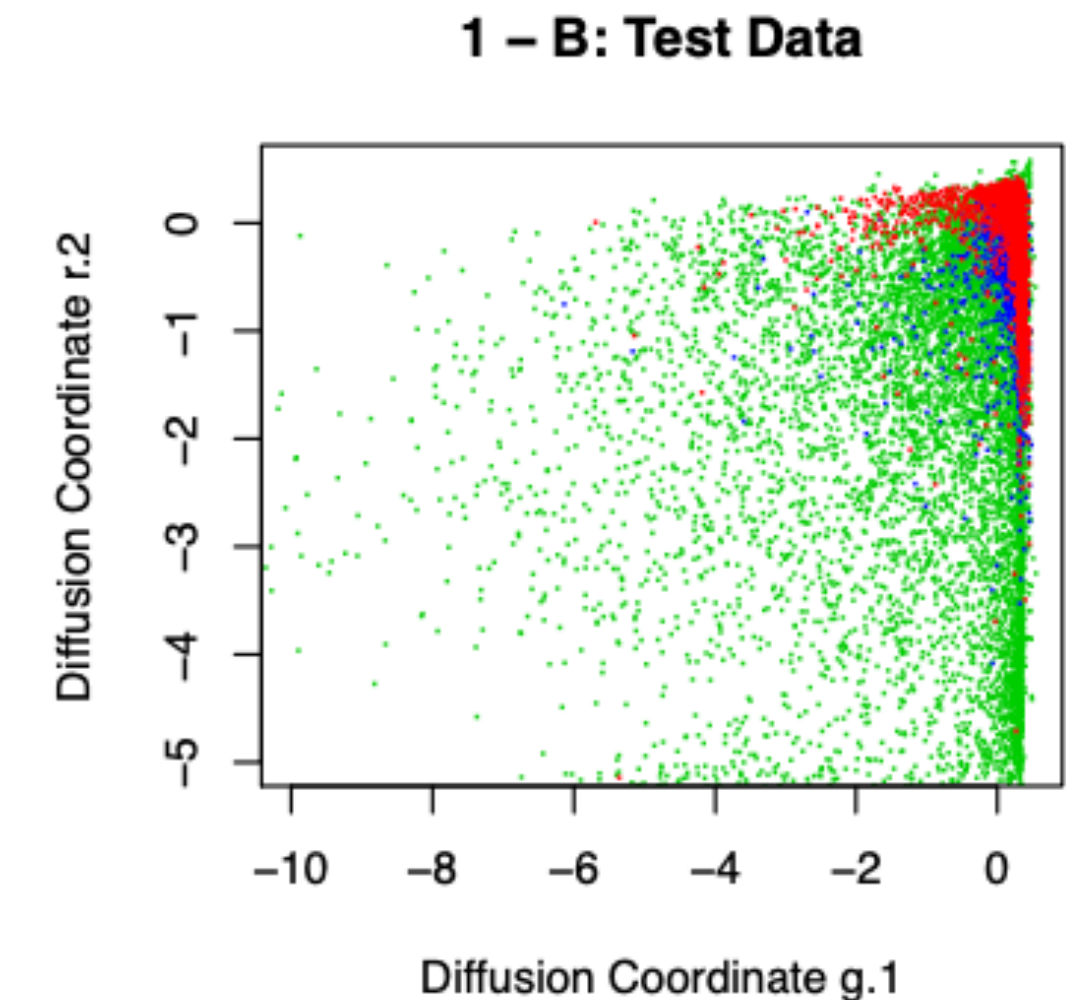
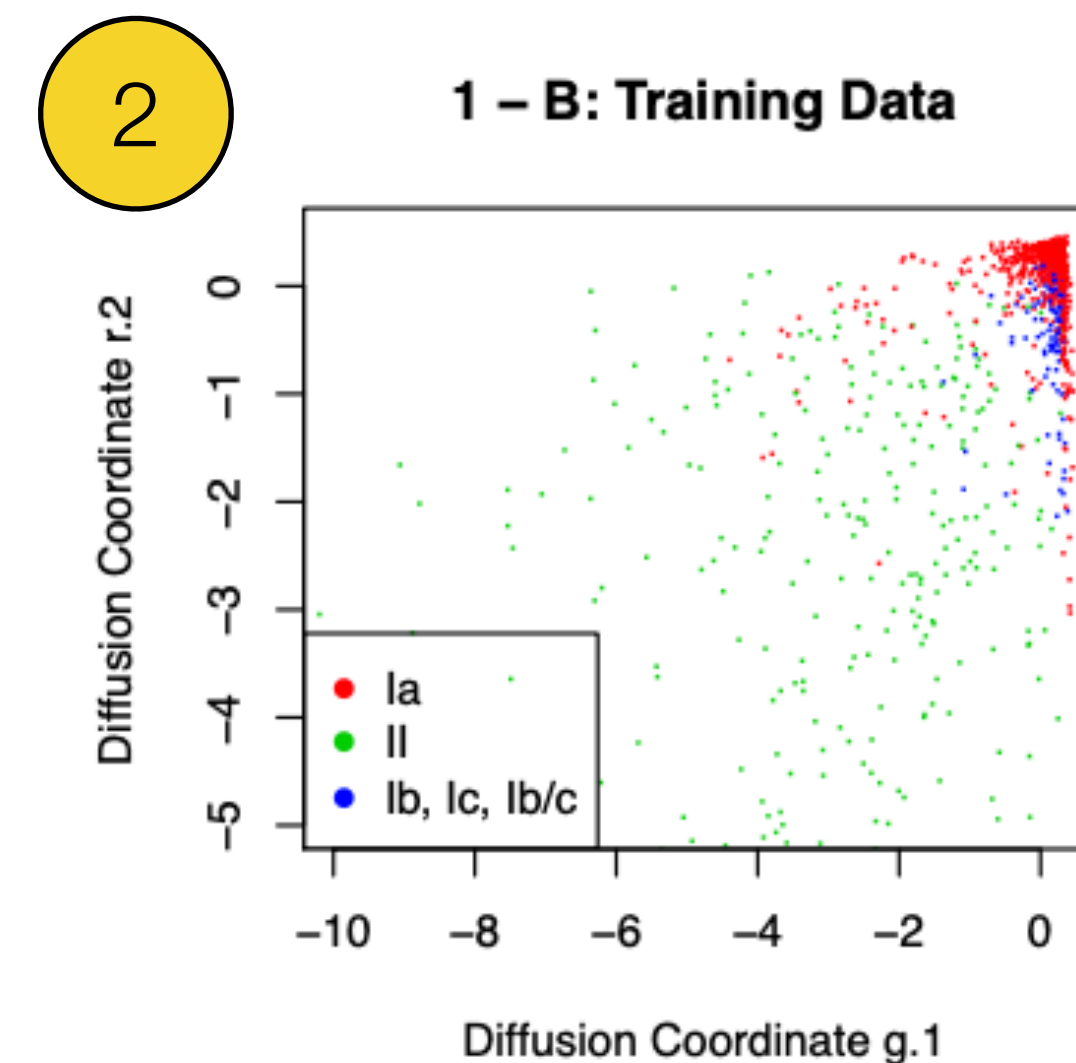
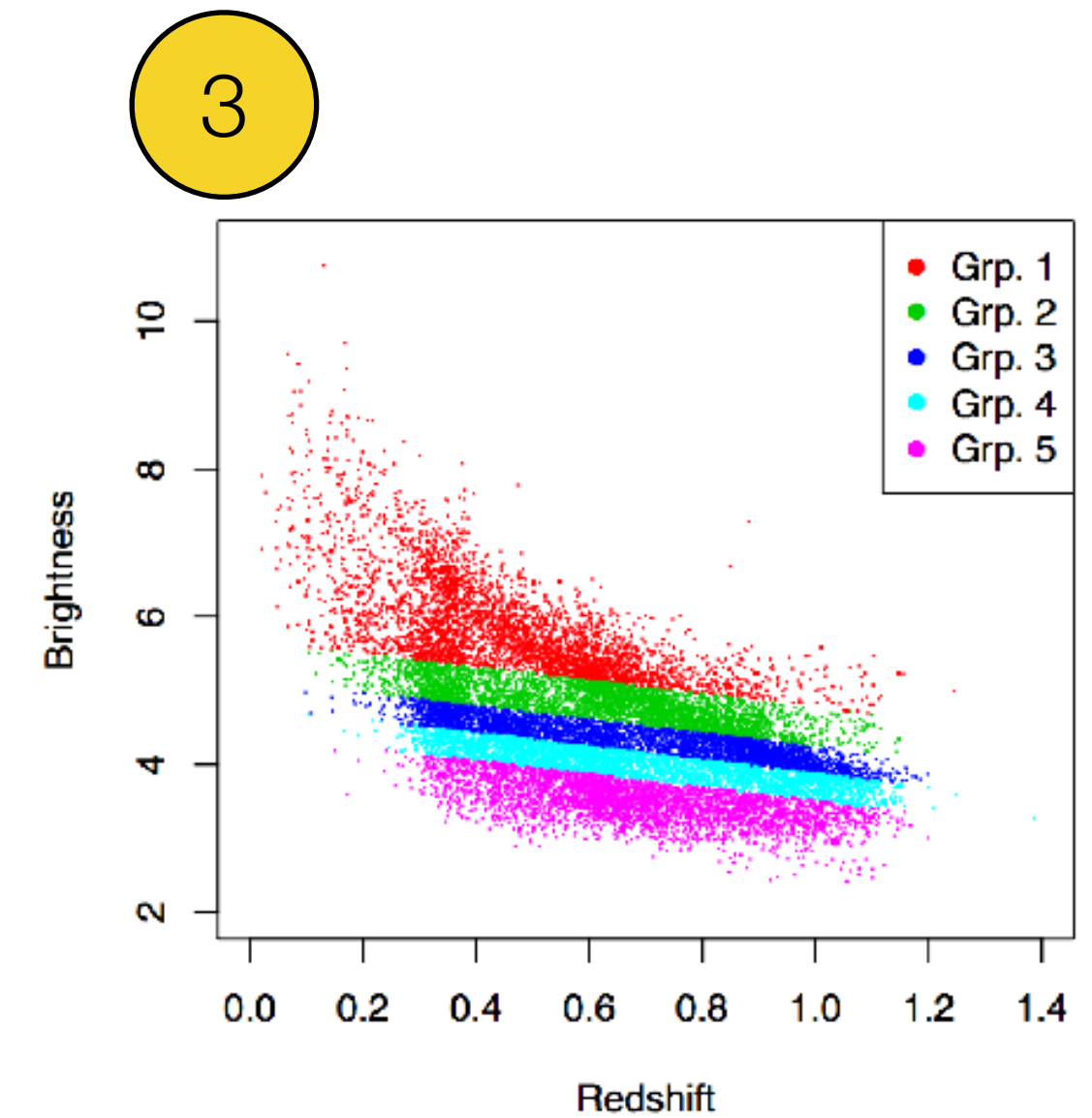
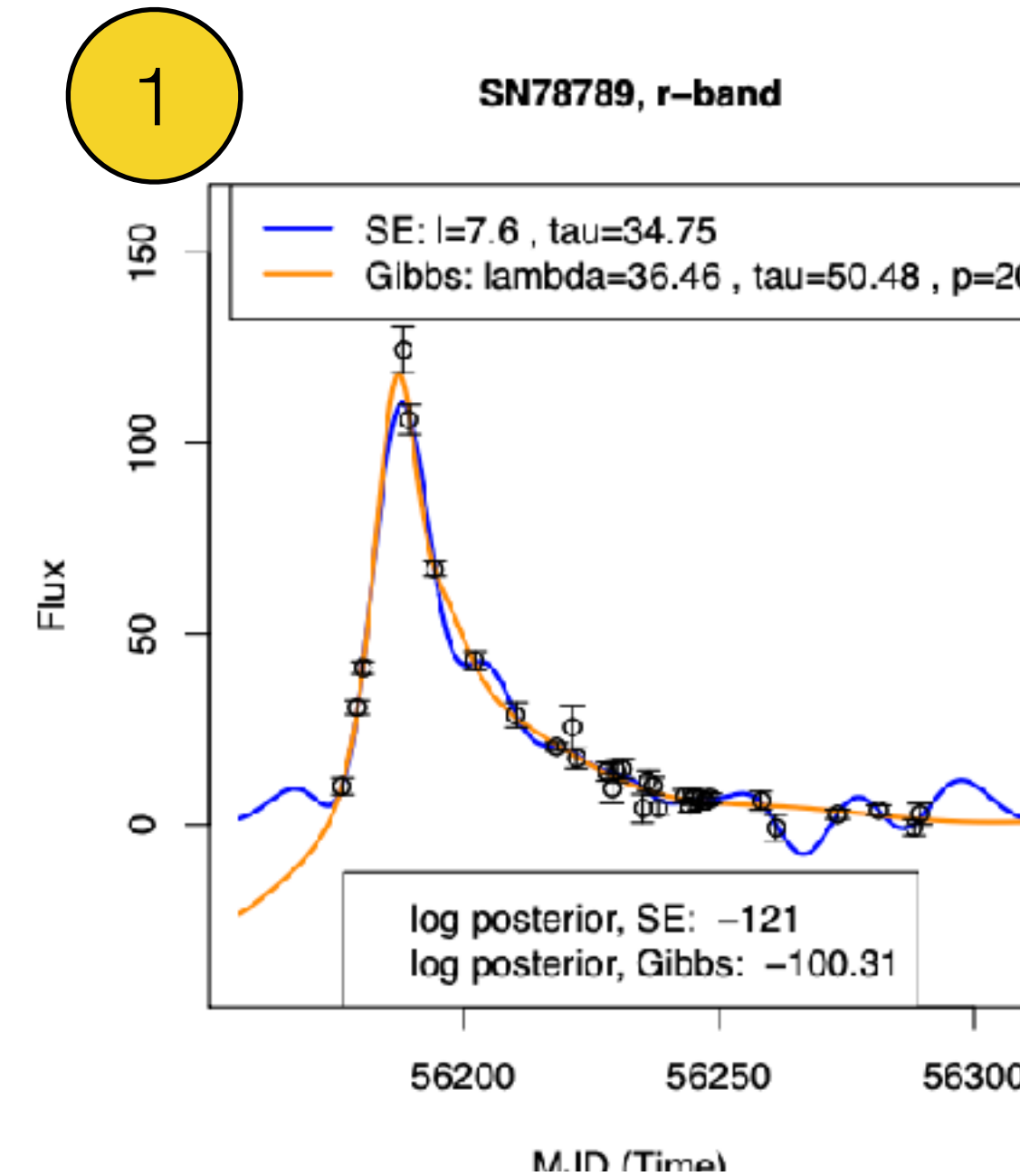


Training set partitioning



STACCATO: Analysis Details

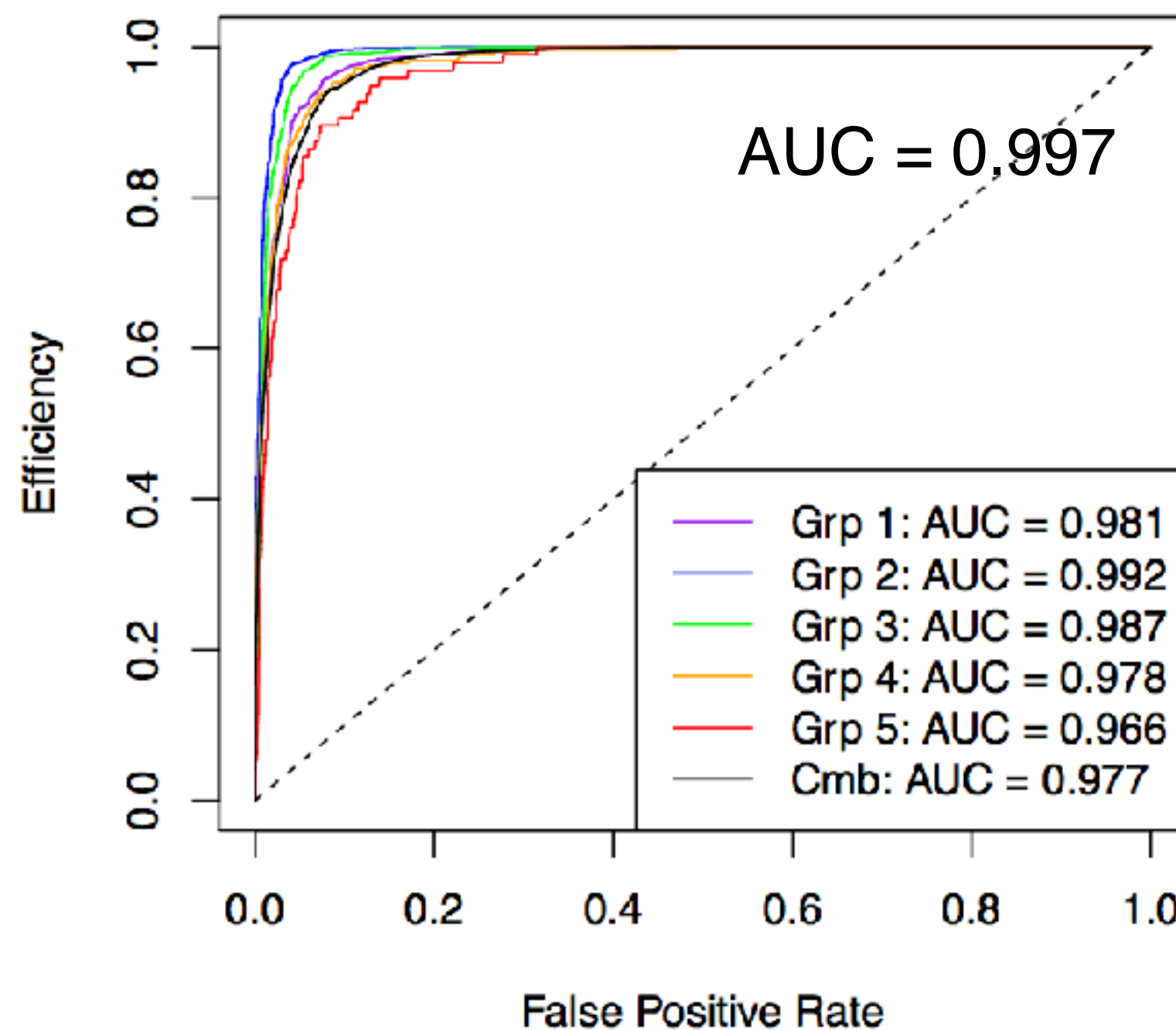
1. Fit Gaussian Process (GP) to light curve data
2. Apply diffusion map technique to map the fitted light curve into a covariate space of $\text{dim} \sim O(100)$, following Richards et al (2012)
3. Stratify light curve data according to propensity scores quantiles
4. Sample new synthetic light curves from fitted GPs in groups with sparse training data (requires validation set to optimise augmentation scheme)
5. Use Random Forest on the stratified diffusion coordinates (group-by-group) for classification



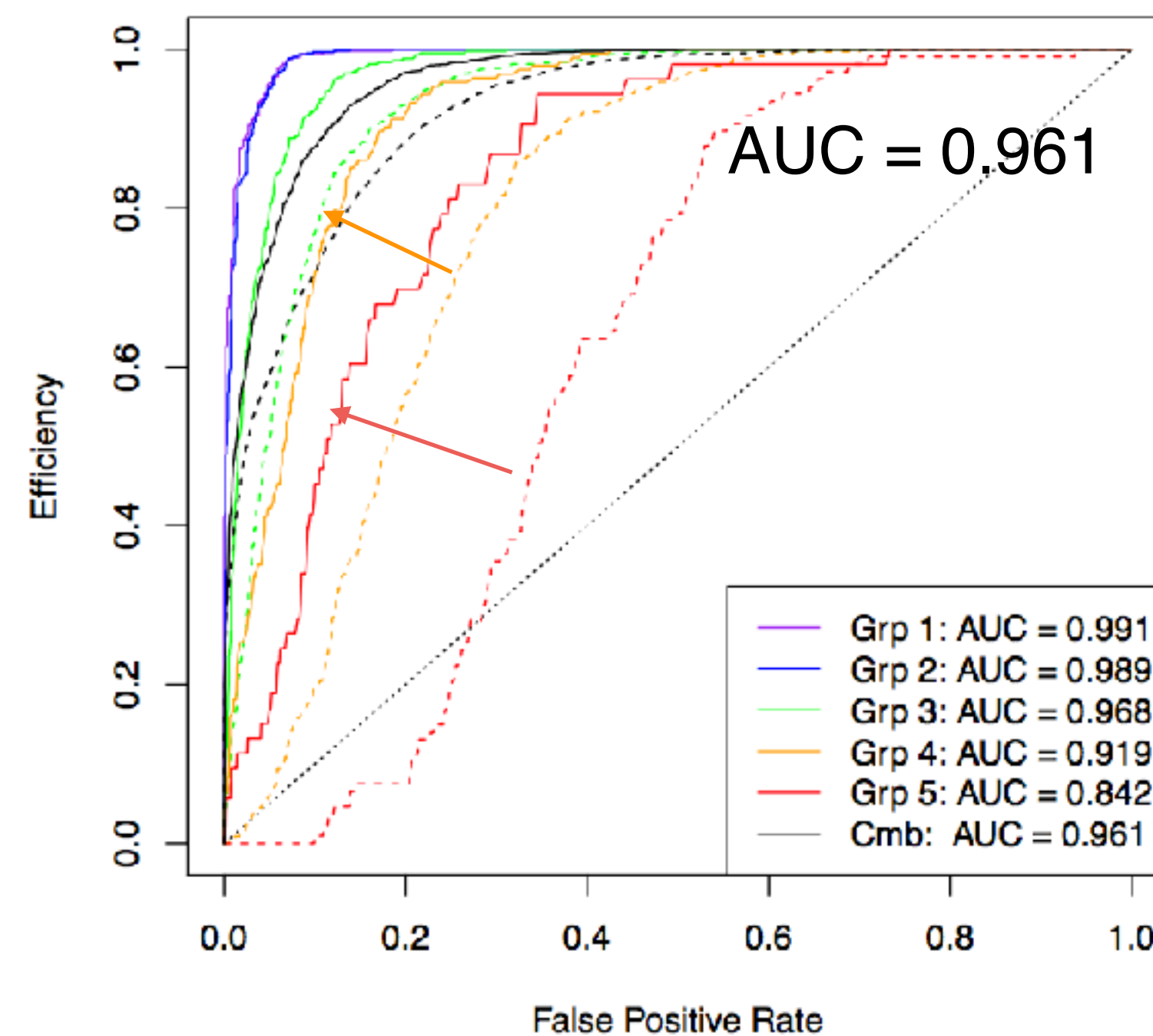
Augmenting LCs via GP Resampling

- STACCATO augments the training set by synthetically generating LCs from the GP **according to the Propensity Scores -> corrects under-sampling where it matters**
- Evaluated using Area under the ROC Curve (AUC)

Gold Standard
(unbiased training set)



STACCATO
dashed (solid) w/o (with) augmentation



Ideally we would like:

A better solution that
does NOT rely on
data augmentation
and its assumptions.

Covariate Shift, or Biased Training Set

Given a feature space, X , and a label space, Y ($K > 1$ classes/dependent variables)
 we have n_s labelled samples $\{x_i^s, y_i^s\}$ from the source domain
 n_t unlabelled samples from the target domain, $\{x_i^t\}$.
 Task: predict $\{y_i^t\}$

Light-curve data

Type Ia or not

Spectroscopic training set

Photometric light-curve only

Is it a Ia?

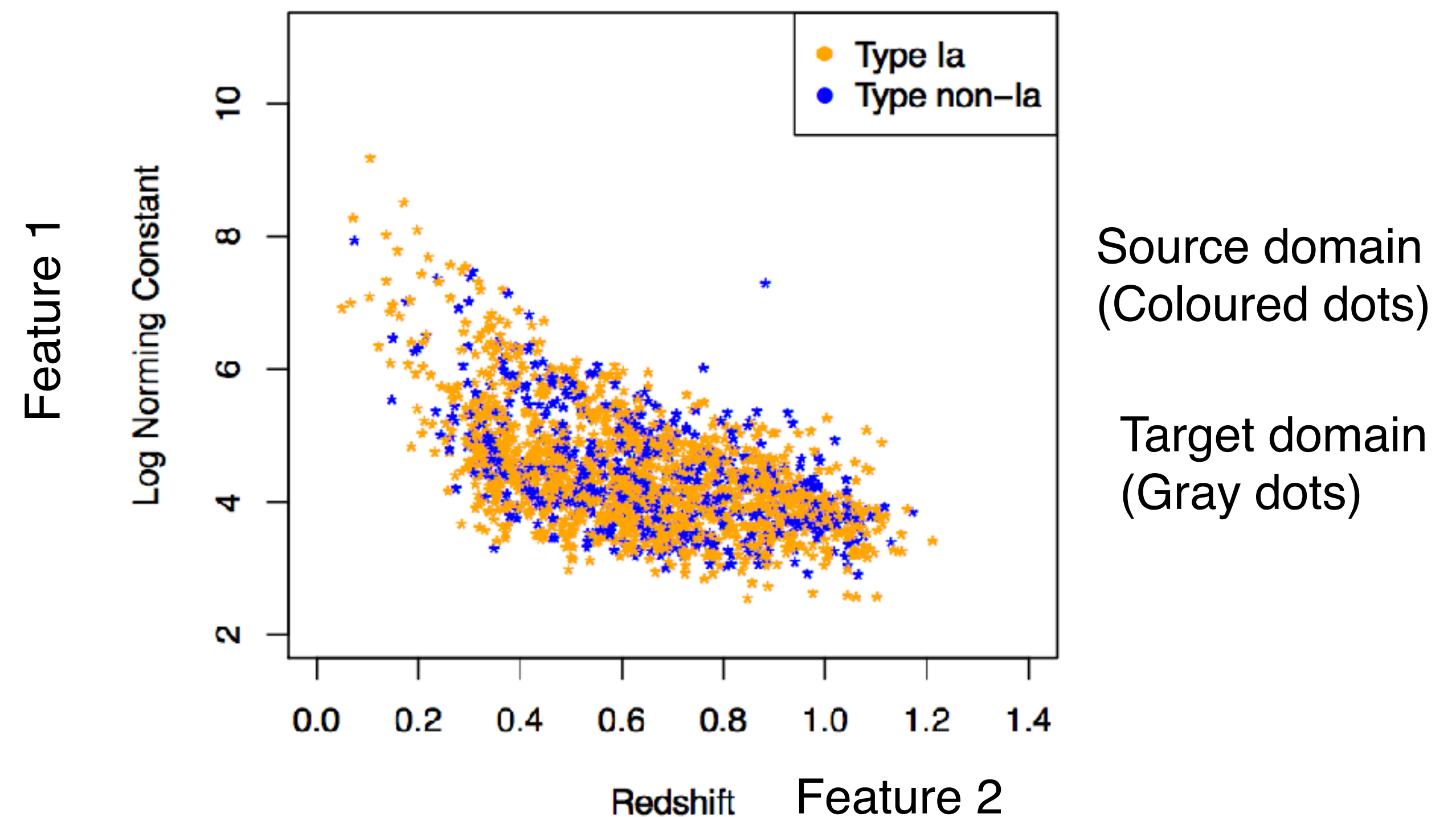
Features: redshift & apparent mag
 Label: Ia or non-Ia

Covariate shift occurs when:

$$p_s(y|x) = p_t(y|x)$$

$$\text{and } p_s(x) \neq p_t(x)$$

I.e., the training set is non-representative of the test set.



Weighted ML estimation of risk

Approach: choose the classification/regression function $f(x)$ so as to minimise the risk (= expected loss) over the target domain.

Shimoidara (2000) showed:

$$E_{(x,y) \sim D_t}[\ell(f(x), y)] = E_{(x,y) \sim D_s} \left[\frac{p_t(x)}{p_s(x)} \ell(f(x), y) \right]$$

Target (i.e. test data) Source (i.e. training data)

The ratio of densities (weights) can be difficult to estimate reliably.

Bias correction approach

Let s be a binary indicator variable controlling training set selection ($s=1$).

Zadrozny (2004) showed:

$$E_{(x,y) \sim D}[\ell(f(x), y)] = E_{(x,y) \sim \tilde{D}} [\ell(f(x), y | s = 1)]$$

$$\tilde{D} = \frac{P(s = 1)}{P(s = 1 | x)} D$$

Estimate $p(s = 1 | x)$ via e.g. logistic regression, then draw samples from $\tilde{D}$.

Our Approach: Propensity Score Stratification

Work by **Max Autenrieth** (Stats PhD student), in collaboration with David van Dyk (Imperial) & David Stenning (Simon Fraser U.)

Improving on our previous work (“STACCATO”), Revsbech, RT, van Dyk (2018): StratLearn, Autenrieth et al (2021), arXiv [2106.11211](https://arxiv.org/abs/2106.11211)

Propensity scores

$e(x_i)$ = probability for object i to be selected into the source domain, using the *whole features* set:

$$e(x_i) \equiv P(s_i = 1 \mid x_s, x_t)$$

Key idea (StratLearn):

subdivide (“stratify”) target and source data in k subgroups according to quantiles of their propensity scores. Then supervised learning in each stratum (“stratified learner”)

Propensity scores as balancing scores

Rosenbaum & Rubin (1983, 1984) show that, conditional on their propensity scores, the k subgroups (“strata”) have approximately balanced covariate distribution, i.e.

$$p_{s_j}(x) \approx p_{t_j}(x) \text{ for } j = 1, \dots, k$$

Since $p_s(y \mid x) = p_t(y \mid x)$, it follows that

$$p_{s_j}(x, y) \approx p_{t_j}(x, y) \text{ for } j = 1, \dots, k$$

Hence covariate shift approximately disappears.

Toy Example (illustration of StratLearn)

1D simulation (regression):

Source: $x_s \sim N(0.5, 0.5^2)$

Target: $x_t \sim N(0.2, 0.5^2)$

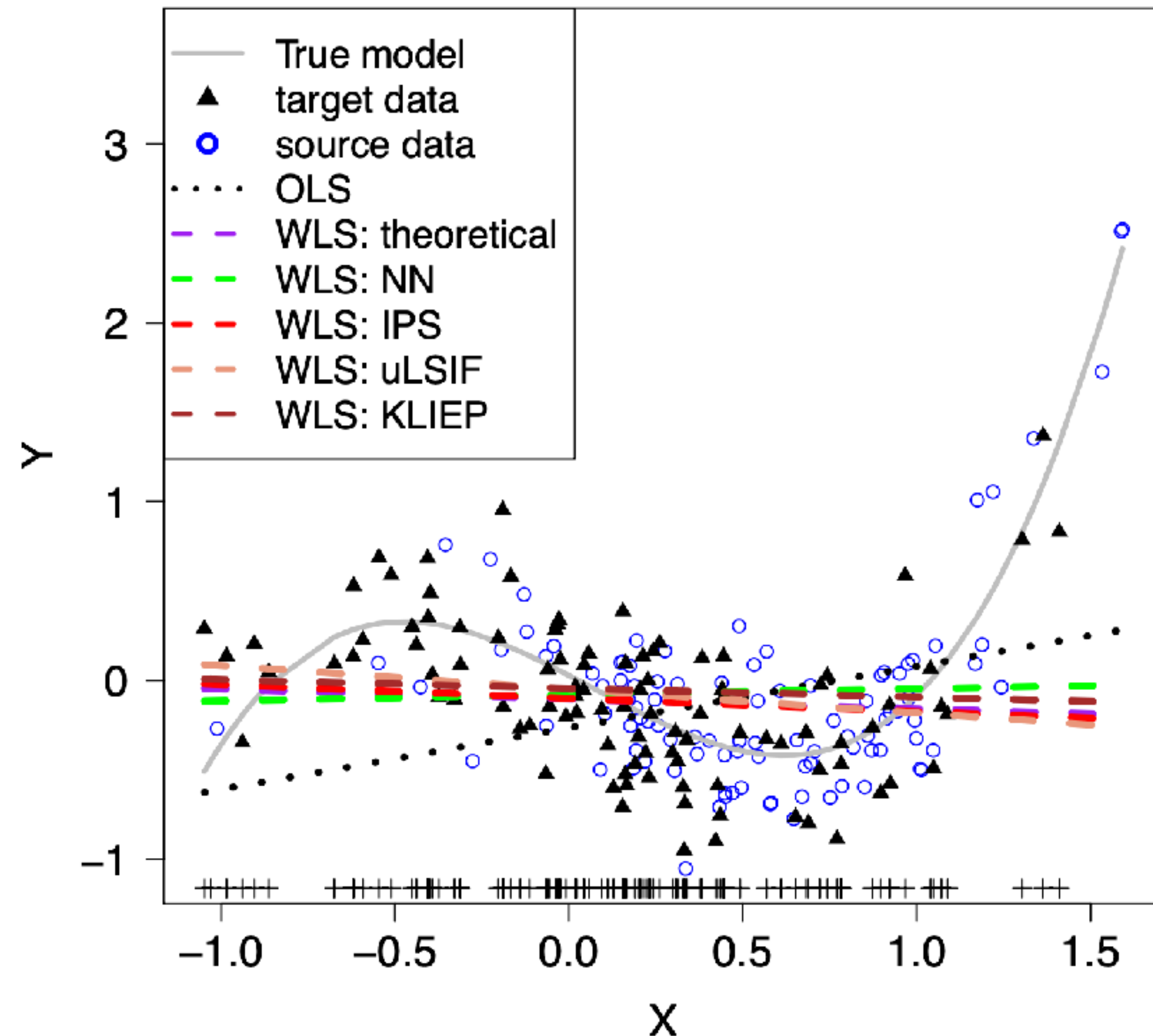
Outcome: $y_s = -x + x^3 + \epsilon$

Where $\epsilon \sim N(0, 0.3^2)$

Fit with misspecified (ie wrong) model:

Ordinary least square regression fit

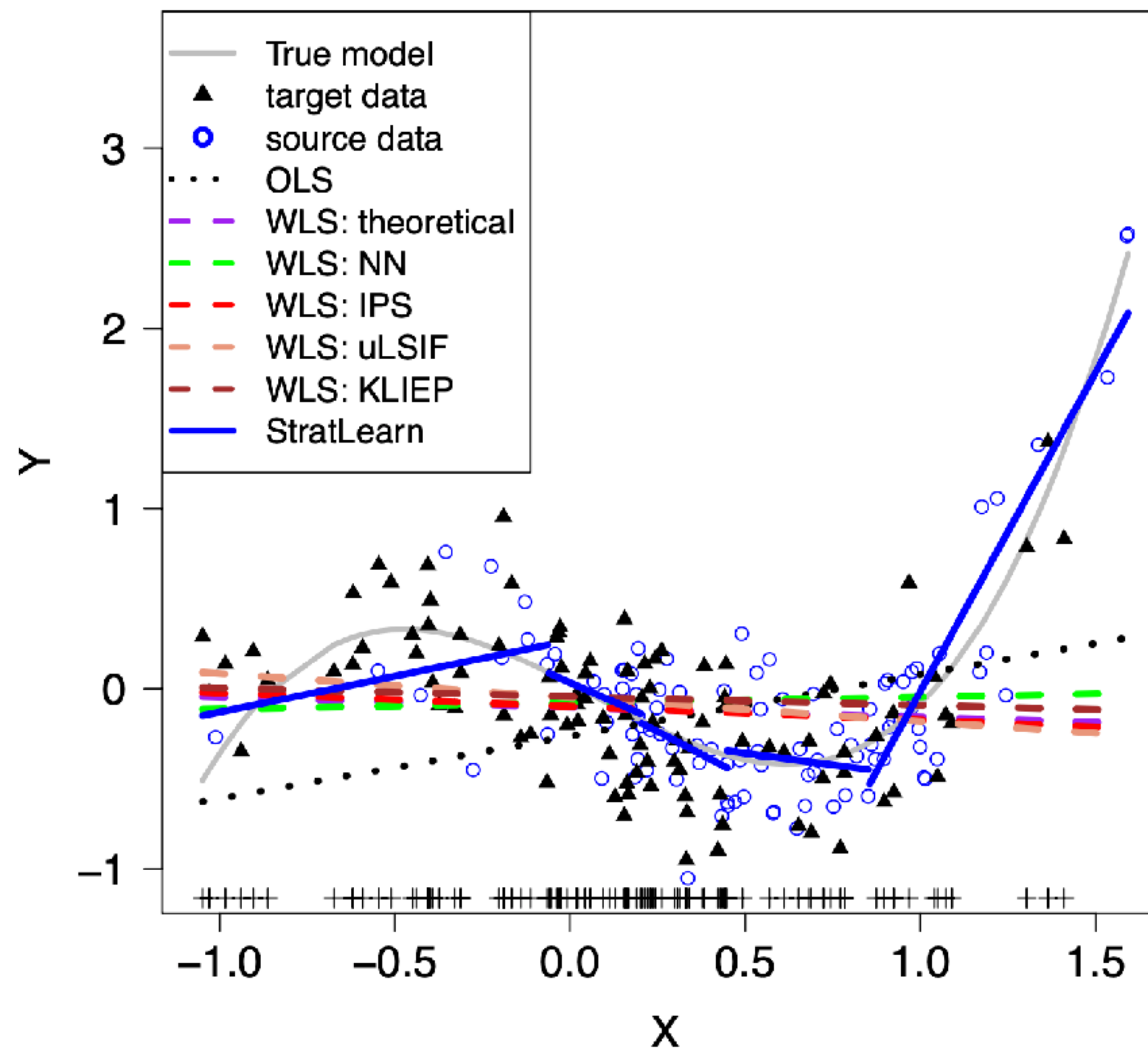
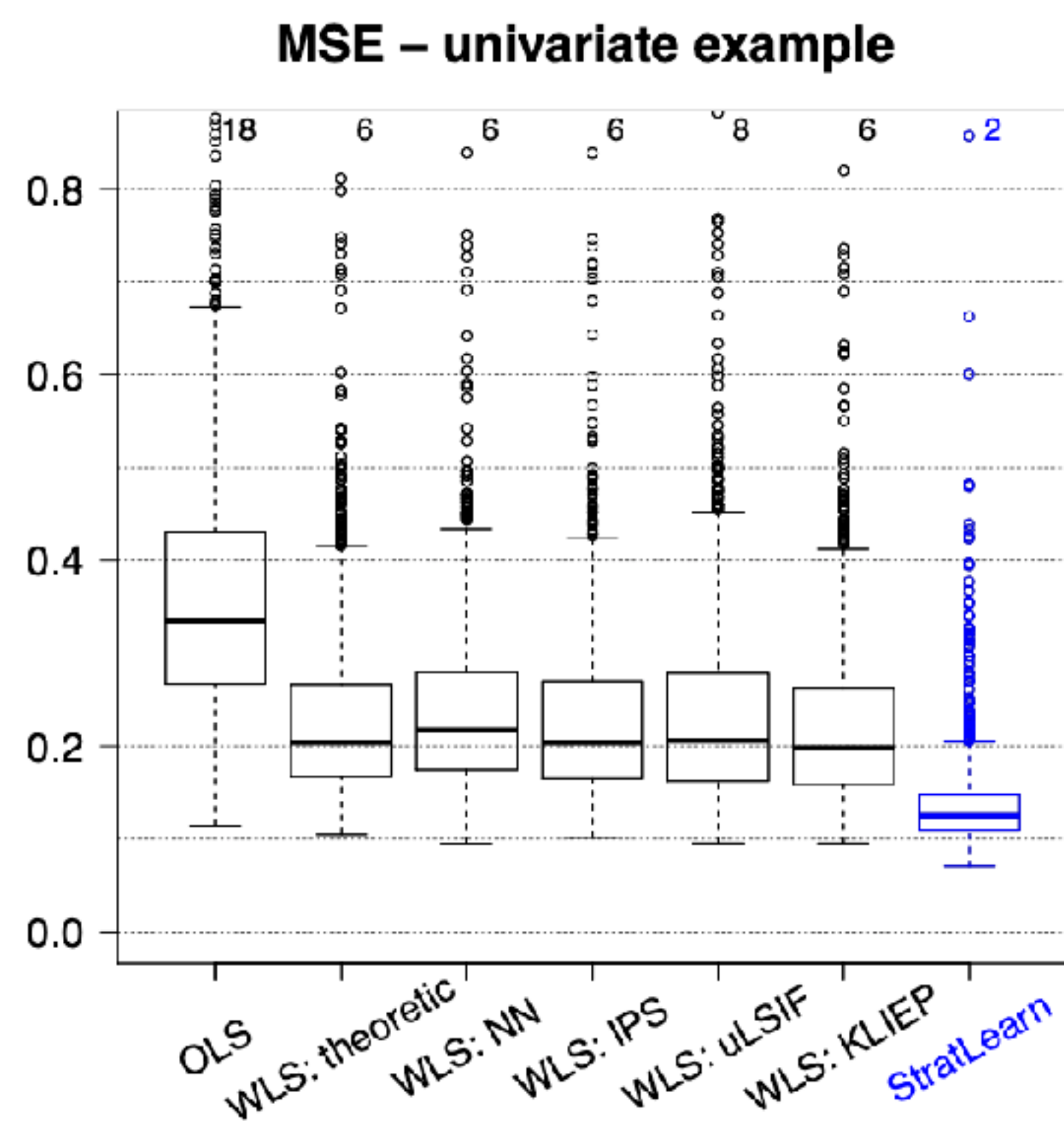
(For the experts: importance weighting does not work in this case)



Toy Example

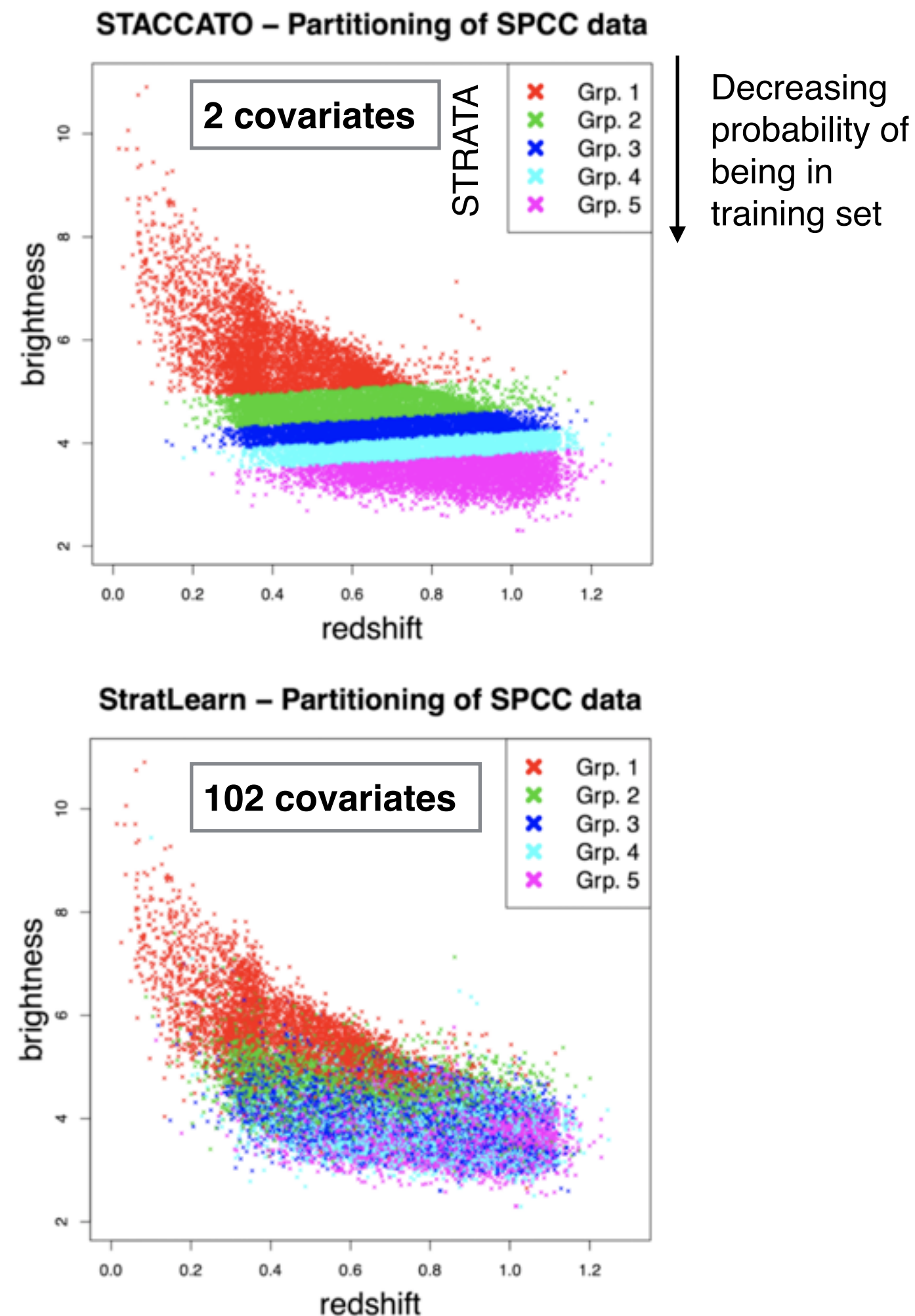
StratLearn solution:

Subdivide the covariate space (ie. x axis) according to quintiles of propensity scores and fit the (wrong) linear model in each



StratLearn on SNIa data

Propensity score partitioning of target domain (test data):



Conditional on the propensity scores (i.e., within each stratum), the source and target outcomes are approximately the same.

This means: inside each stratum, the imbalance has been redressed, i.e. source data are approximately representative

Important: the underlying theorem only valid *if all potential confounding covariates* (i.e., things the SNIa type could depend on) are included in the propensity score estimation!

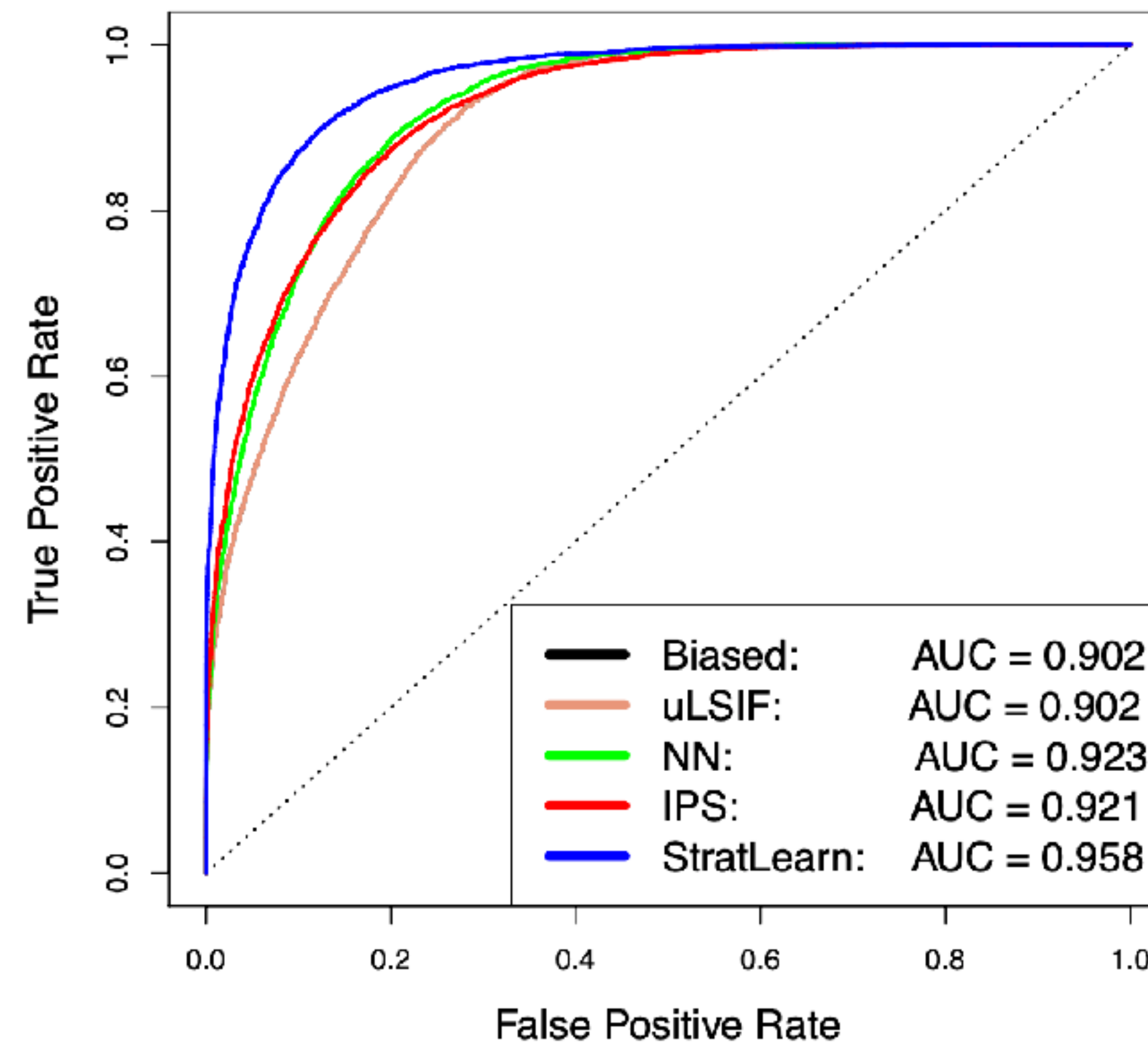
Stratum	Set	Number of SNe	Number of SNIa	Prop. of SNIa
1	Source	958	518	0.54
	Target	3306	1790	0.54
2	Source	120	28	0.23
	Target	4144	927	0.22
3	Source	13	4	0.31
	Target	4250	540	0.13
4	Source	7	4	0.57
	Target	4257	610	0.14
5	Source	4	4	1
	Target	4259	662	0.16

👍 Balanced proportions

👍 Balanced proportions

Classification Performance with StratLearn

SPCC Challenge data (v2):



StratLearn performance close to “gold standard” of unbiased training set (AUC=0.977 vs 0.958) without any augmentation

Cf previous results:
Lochner et al (2016):
AUC= 0.855

Pasquet et al (2019):
AUC=0.939

Revsbech et al
 (“STACCATO”, 2018):
AUC=0.94

Note: AVOCADO (Boone, 2019), winner of the PLASTiCC challenge 2019, uses an extended version of STACCATO (incl. augmentation).

StratLearn for regression

We seek a principled framework for covariate shift adaptation via propensity score stratification that does not need augmentation:

1. Augmentation is problem-specific
2. Augmentation usually requires a validation set (not available)

Propensity score stratification leads to approximately balanced (i.e., unbiased) sub-groups, on which to perform supervised classification/regression.

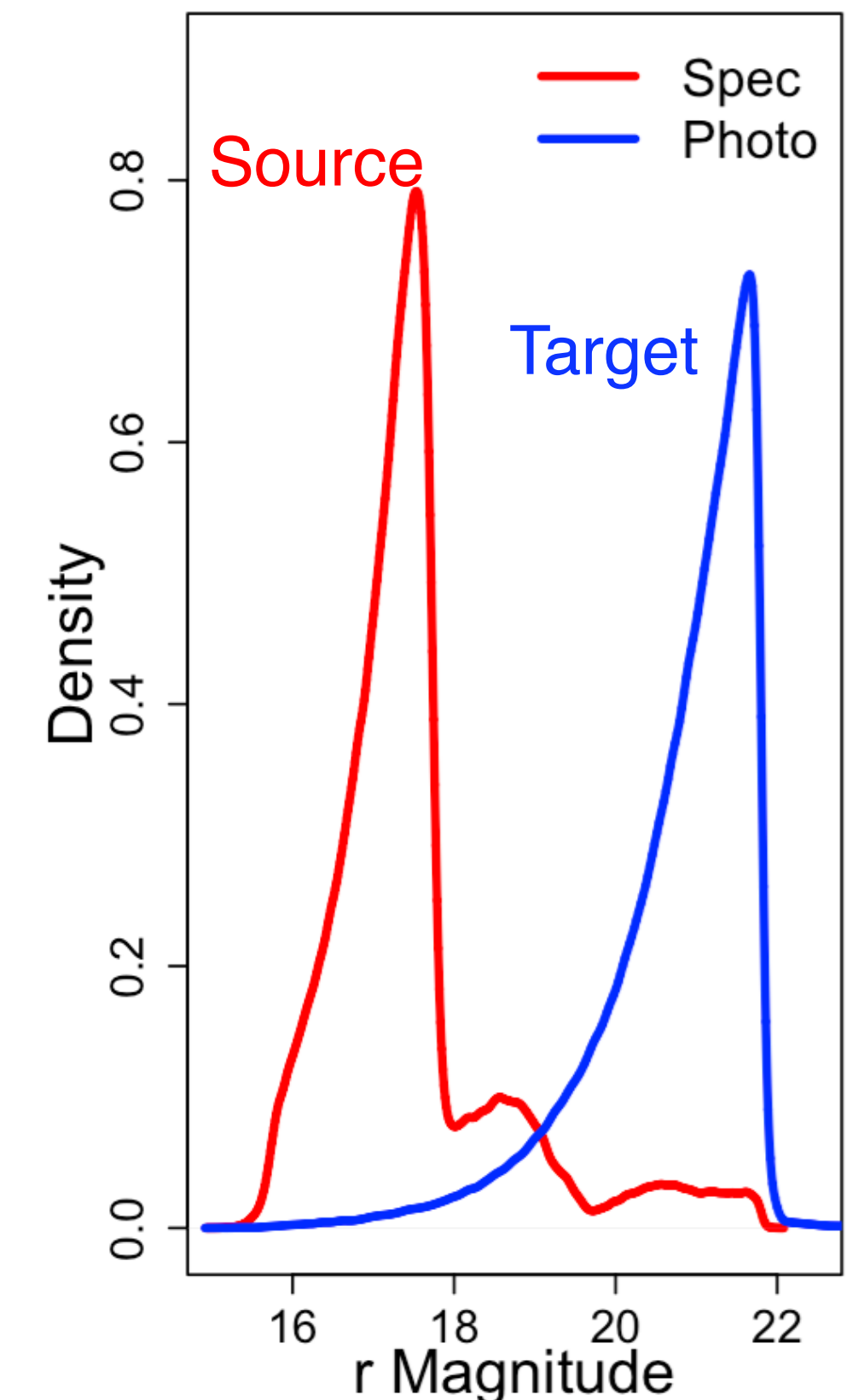
Classification (2-way)

SN Ia vs non-Ia, AUC:

Gold standard: 0.977
Best-in-class : 0.937
STACCATO : 0.961
StratLearning : **0.973**

Multivariate regression

Photo-z estimation
(~500,000 source/targets from
SDSS DR 8)



Conditional Density Estimation Problem

The problem: estimate the conditional density of redshift, z , given the observed covariates (magnitudes in 5 different colour filters), in the presence of covariate shift.

We compare our performance to the estimators in Izbicki et al (2017).

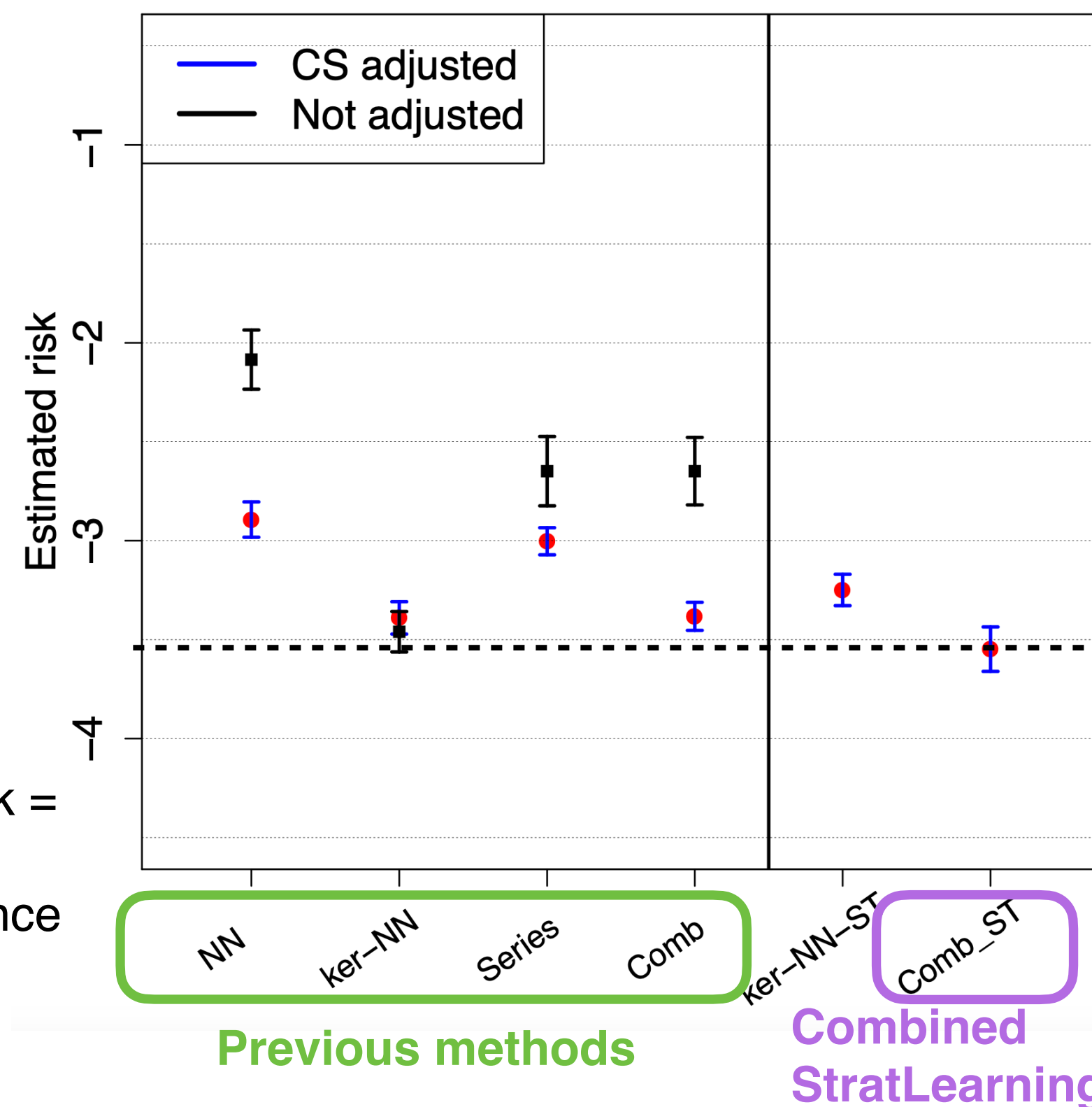
Approach:

1. “StratLearning” partitions the source and target domain according to propensity scores (**no augmentation needed**)
2. Within each group, we combine two conditional density estimation models from Izbicki et al (2017), *ker-NN* and *Series*, via a weighted average.
3. Weight is optimised by minimising the empirical loss on a validation set (a sub-set of the training set, **no test data needed**)

StratLearn: Photo-z Performance

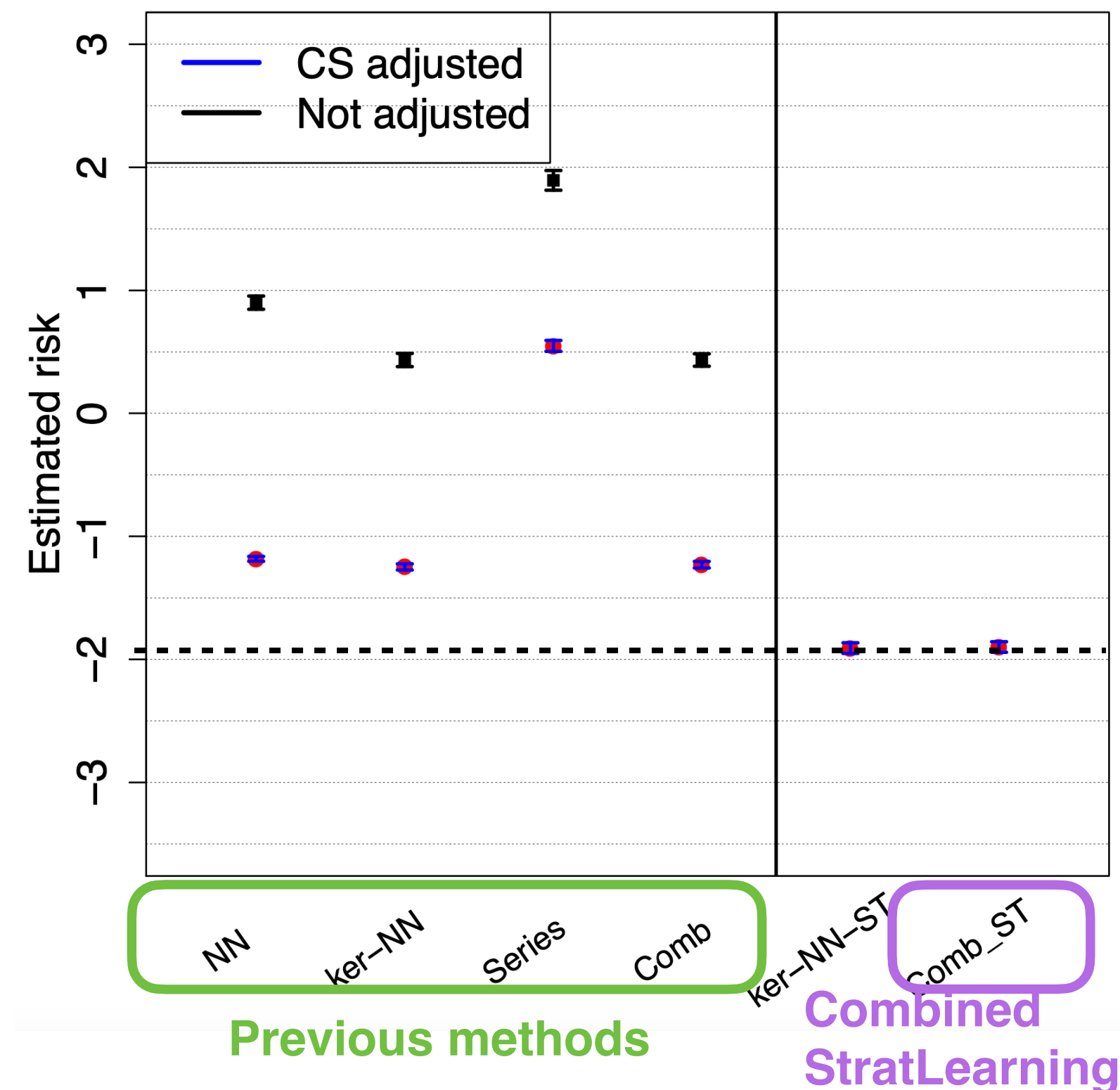
Moderate shift
Low covariate dimensions

Target loss: Beta(13,4)



Moderate shift
High covariate dimensions

Target loss: Beta(13,4) + 50 noise covariates



StratLearning
outperforms
previous
methods for this
problem.

Performance
improvement is
larger in the
presence of
high-D noisy
covariate space
(right panel).

- 1 Covariate shift is an important and recurrent phenomenon in supervised learning. In dark energy research, it will affect the next generation of large SNIa data.
- 2 We propose a general approach (StratLearn) based on stratifying source and target domain according to propensity scores (= probability of an object to be included in the source domain).
- 3 Within strata, source and target domains are better balanced: StratLearn shows improved performance in regression and classification tasks compared to best-in-class alternatives.

Thanks to my collaborators: Max Autenrieth (PhD student), David van Dyk (Imperial), David Stenning (Simon Fraser U.). Paper here: <https://arxiv.org/abs/2106.11211>

Thank you!

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