Bayesian inference
of
astronomical populations

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Astronomical data

Astronomical samples

- Astronomical survey records data from the observable sky.
- Sample is a list/catalogue of objects "detected" in a survey (effectively data compression).
- Each has (noisy) measurements of observable properties.
- Possible sample analysis tasks:
    - Infer properties of individual objects.
    - Classify individual objects.
    - Constrain population from which sample is drawn.
    - Identify correlations, *etc.*, at the population level.

Bayesian inference

- One basic aim of science: ascribe probabilities to models/hypotheses given data/knowledge.
- Requires probability is defined as a degree of implication, hence $P(A|B)$ notation.
- Cox (1946) showed that logical and self-consistency implies (equivalence with) the standard laws of probability.
- Hence want (posterior) probabilities obtained by applying Bayes's theorem and other laws of probability to data-sets.
- Methodology prescriptive; choice is in models.

Population model

- Poisson point process (but could be binomial, cf Kelly *et al.* 2008) with density:

population parameters (normalisation, slope, *etc.*)

$$\mathrm{d}\bar{N}_t = \rho_t(\boldsymbol{\phi};\boldsymbol{\psi})\,\mathrm{d}\boldsymbol{\phi}$$

type

object parameters (mass, luminosity, *etc.*)

- Similar to, *e.g.*, Loredo (2004) and does not need to be normalisable (cf Buchner *et al.* 2015).
- Common examples:
  - Schechter (1976) luminosity function (but really just a gamma distribution)
  - Salpeter (1955) stellar mass function (but really just a Pareto/power-law distribution)

Observation model

- Measurement process encoded in source sampling distribution or source likelihood:

$$\mathrm{P}(\boldsymbol{d}|t, \boldsymbol{\phi}, \mathcal{O})$$

- Selection into the catalogue has the form:

$$\mathrm{P}(S|\boldsymbol{d}, t, \boldsymbol{\phi}, \mathcal{O})$$

- Loredo (2004) emphasises deterministic selection, but if only data summaries (e.g., fluxes, positions) are available then it is effectively probabilistic (cf Buchner *et al.* 2015).

Population/object inference

$$\mathrm{P}(\boldsymbol{\psi}, t_{1:N}, \boldsymbol{\phi}_{1:N} | N, \boldsymbol{d}_{1:N}, S_{1:N}, T, \mathcal{O}, \mathcal{K})$$

$$\propto \mathrm{P}(\boldsymbol{\psi}|T, \mathcal{K}) \, \frac{\mathrm{P}(N|\boldsymbol{\psi}, T, \mathcal{O})}{[\bar{N}(\boldsymbol{\psi}, T, \mathcal{O})]^N} \prod_{i=1}^{N} \rho_{t_i}(\boldsymbol{\phi}_i; \boldsymbol{\psi}) \, \mathrm{P}(\boldsymbol{d}_i | t_i, \boldsymbol{\phi}_i, \mathcal{O}) \, \mathrm{P}(S_i | \boldsymbol{d}_i, t_i, \boldsymbol{\phi}_i, \mathcal{O})$$

where

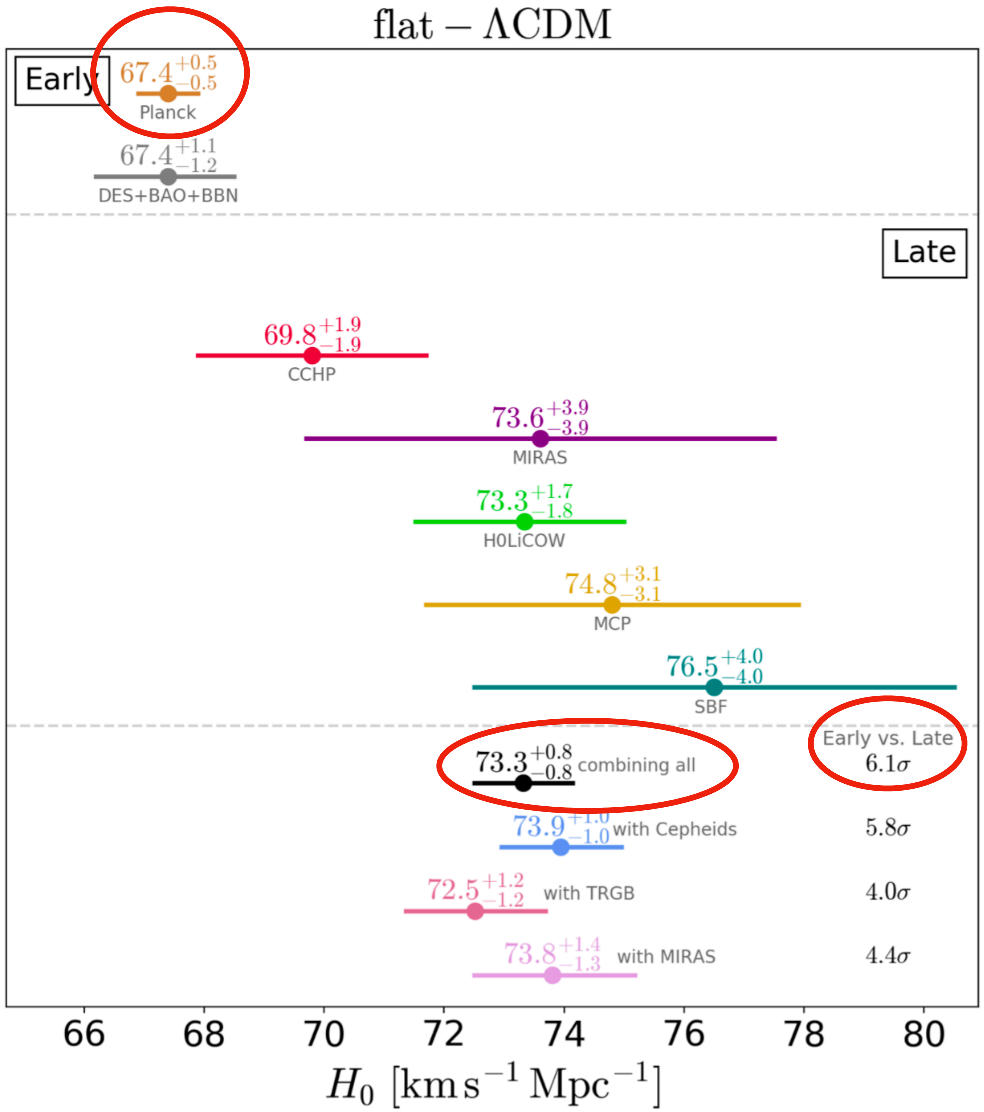
$$\bar{N}(\boldsymbol{\psi}, T, \mathcal{O}) = \sum_{t=1}^{T} \int \mathrm{d}\boldsymbol{\phi} \, \rho_t(\boldsymbol{\phi}; \boldsymbol{\psi}) \, \mathrm{P}(S|t, \boldsymbol{\phi}, \mathcal{O})$$

- Does not include:
  - false detections
  - clustering or inter-object correlations
  - source crowding
  - selection function estimation

The value of Hubble constant

with:
Stephen Feeney and Niccolo D’almasso

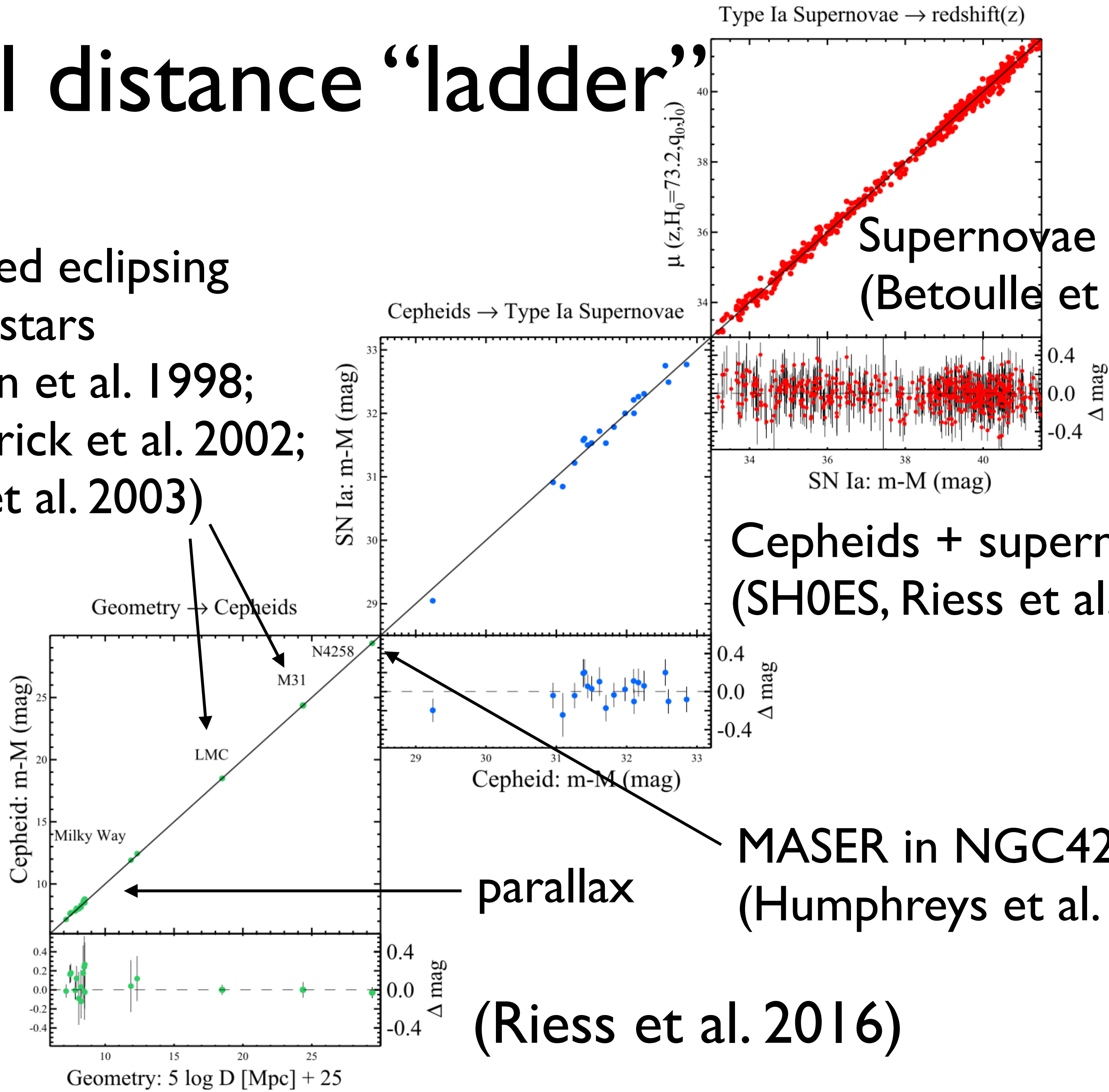
Hubble trouble?



(Verde et al. 2019)

Local distance “ladder”

detached eclipsing binary stars (Guinan et al. 1998; Fitzpatrick et al. 2002; Ribas et al. 2003)



Supernovae (Betoulle et al. 2014)

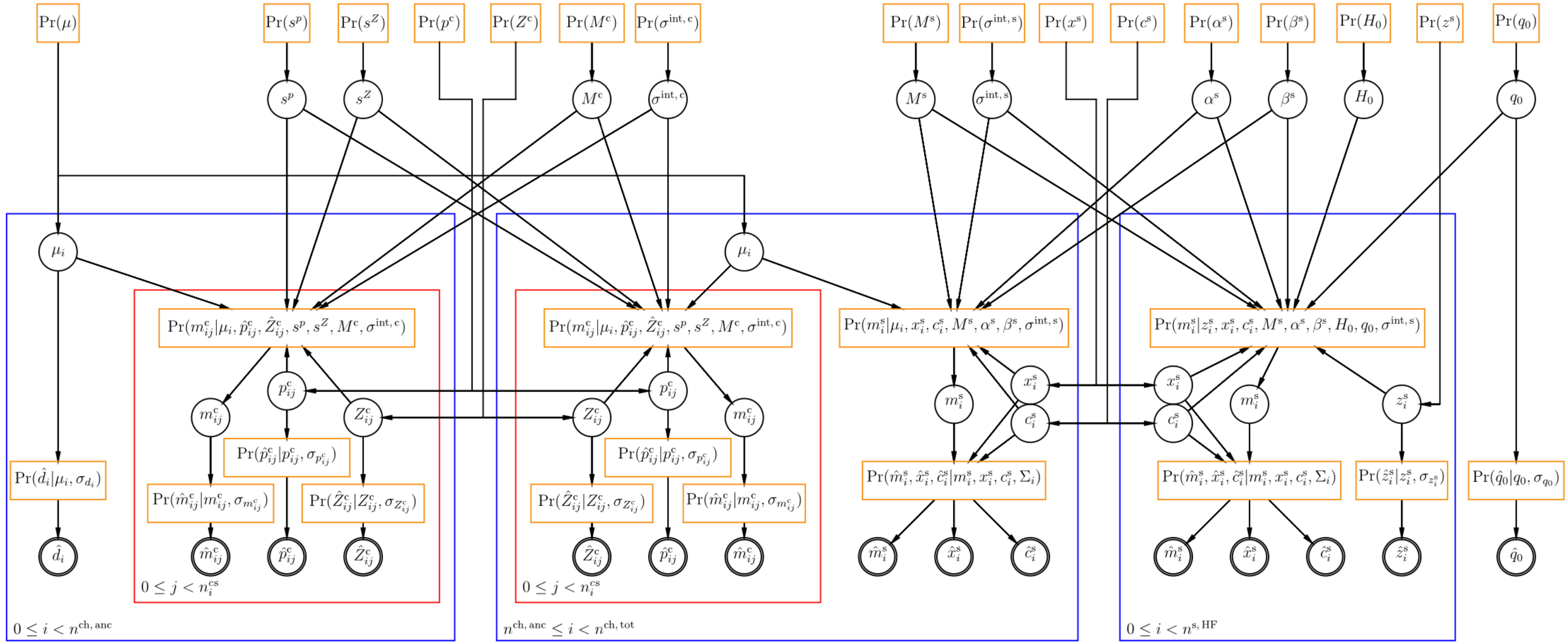
Cepheids + supernovae (SH0ES, Riess et al. 2016)

MASER in NGC4258 (Humphreys et al. 2013)

parallax

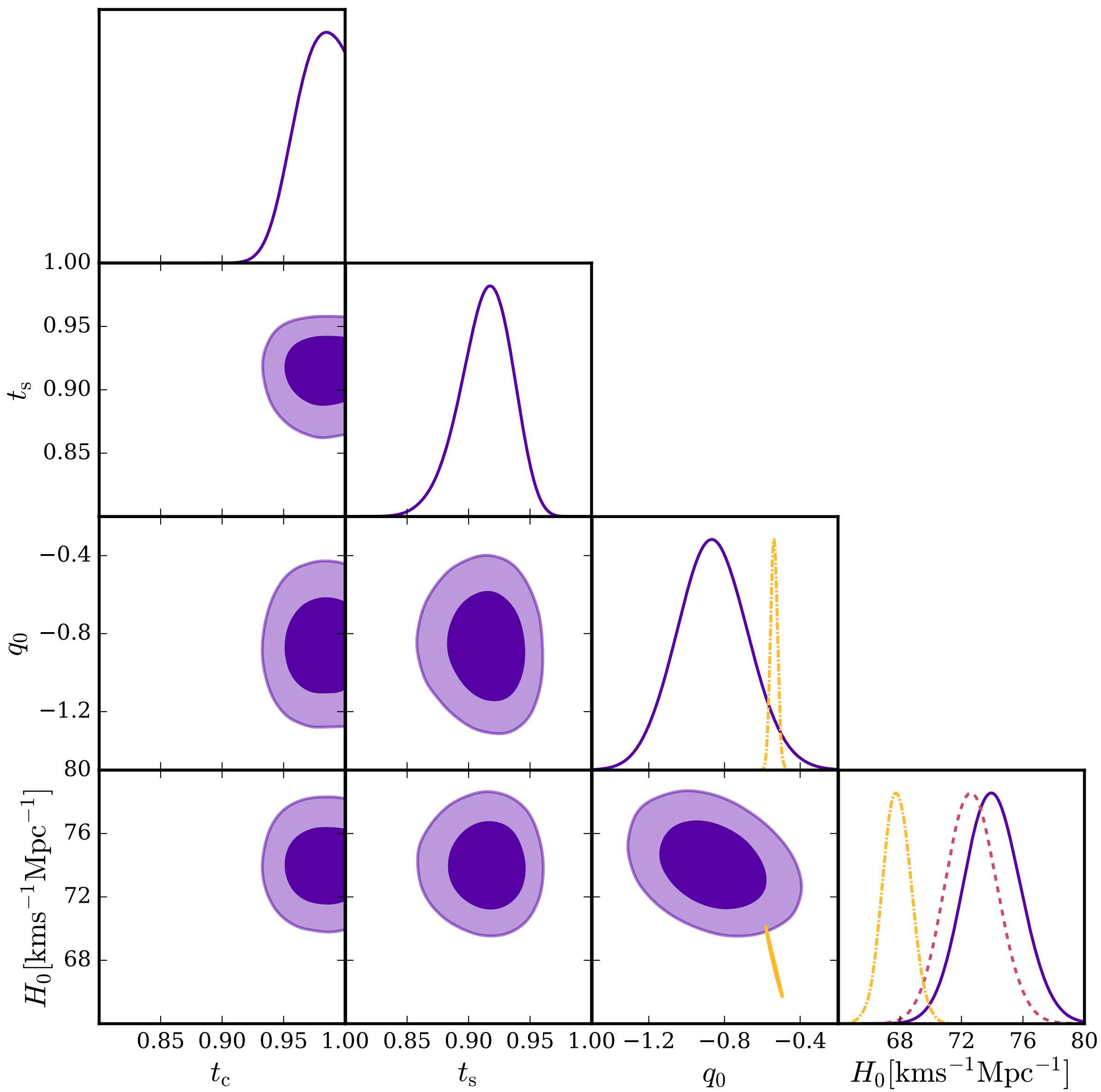
(Riess et al. 2016)

Bayesian hierarchical model



(Feeney et al. 2018)

Results



1.00
0.95
0.90
0.85
t_s
−0.4
−0.8
−1.2
q_0
80
76
72
68
$H_0[\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}]$
0.85 0.90 0.95 1.00 0.85 0.90 0.95 1.00 −1.2 −0.8 −0.4 68 72 76 80
t_c
t_s
q_0
$H_0[\mathrm{kms}^{-1}\mathrm{Mpc}^{-1}]$

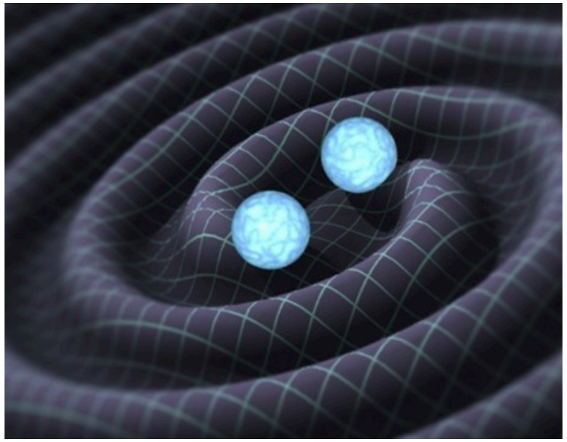
Cosmology with compact merger gravitational wave counterparts

currently with:
Hiranya Peiris (UCL & Stockholm University)
Samaya Nissanke (GRAPPA, Amsterdam)
Nikhil Sarin (Nordita, Stockholm University)
Justin Alsing (Stockholm University)

previously with:
Stephen Feeney & Andrew Williamson

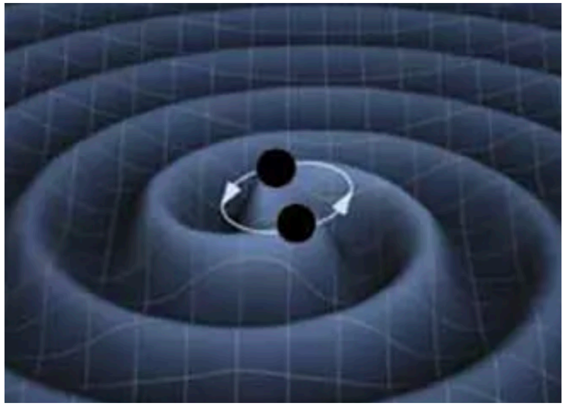
Compact binary mergers

Binary neutron stars:
- *M* ~ Solar mass
- *R* ~ 10 km
- optical counterpart from kilonova or gamma-ray burst



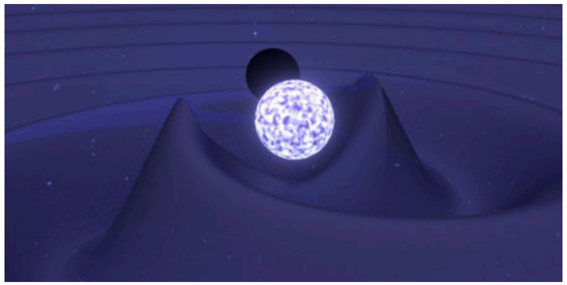
(Caltech/LIGO)

Binary black holes:
- *M* ~10 Solar mass
- *R* ~ 100 km
- no expected optical emission



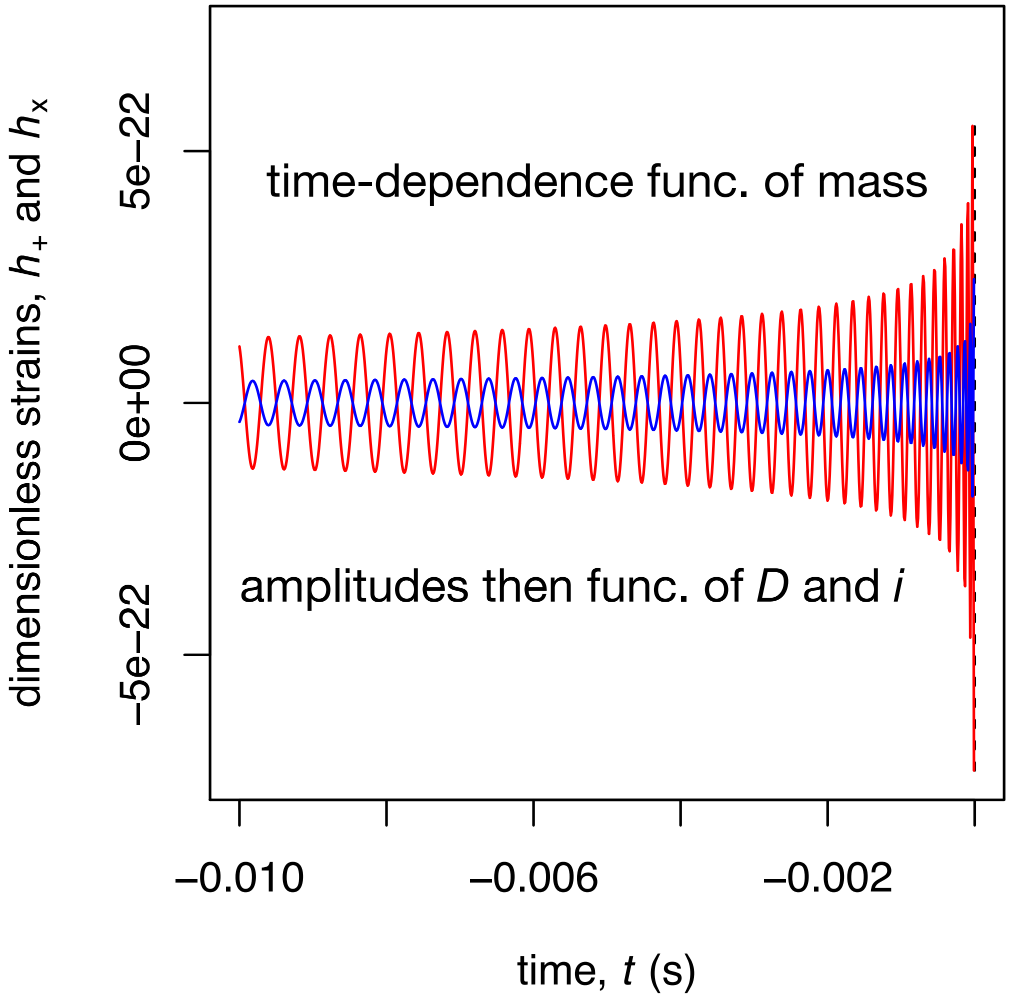
(Caltech/LIGO)

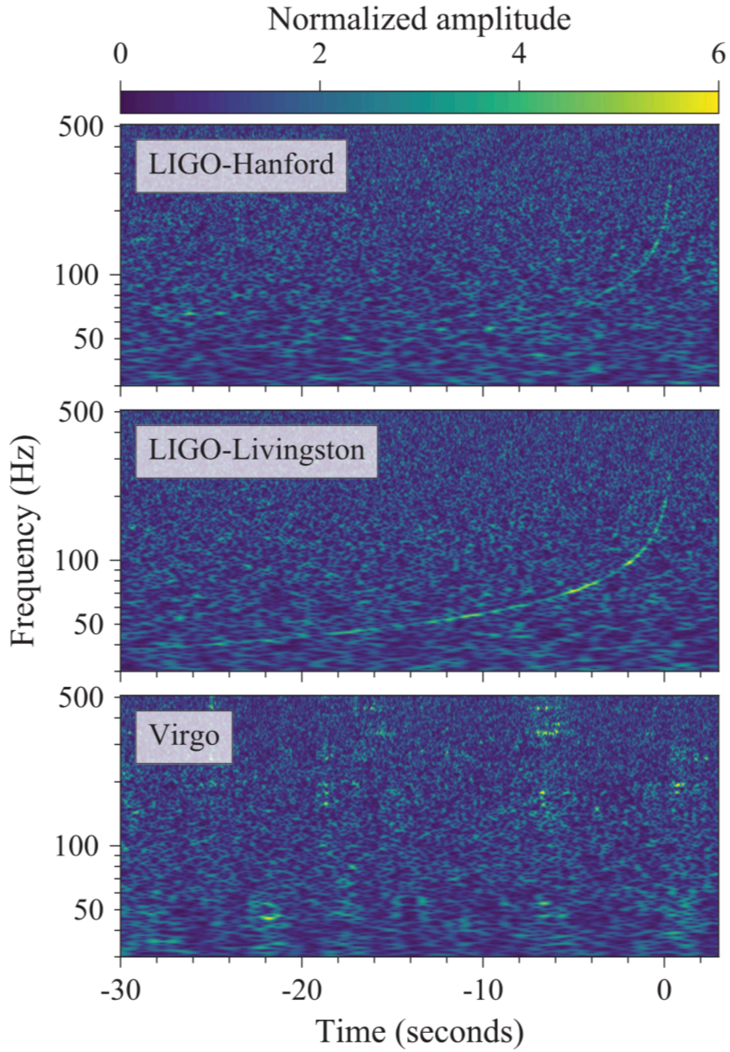
NS-BH mergers:
- properties less certain



(OzGrav)

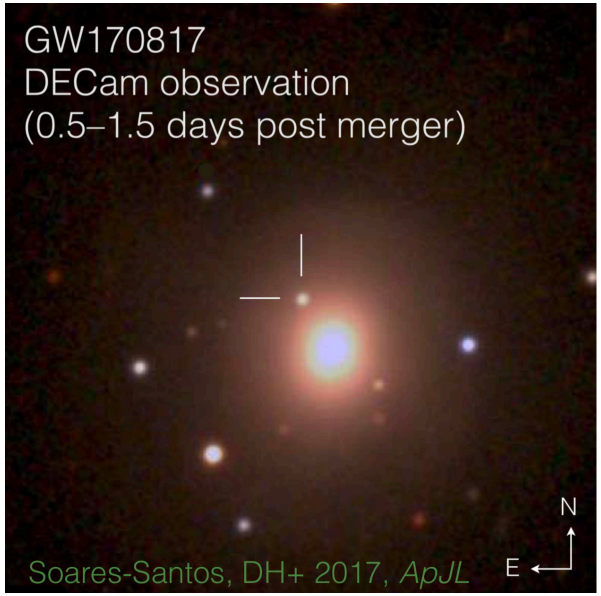
GWs from a BNS merger





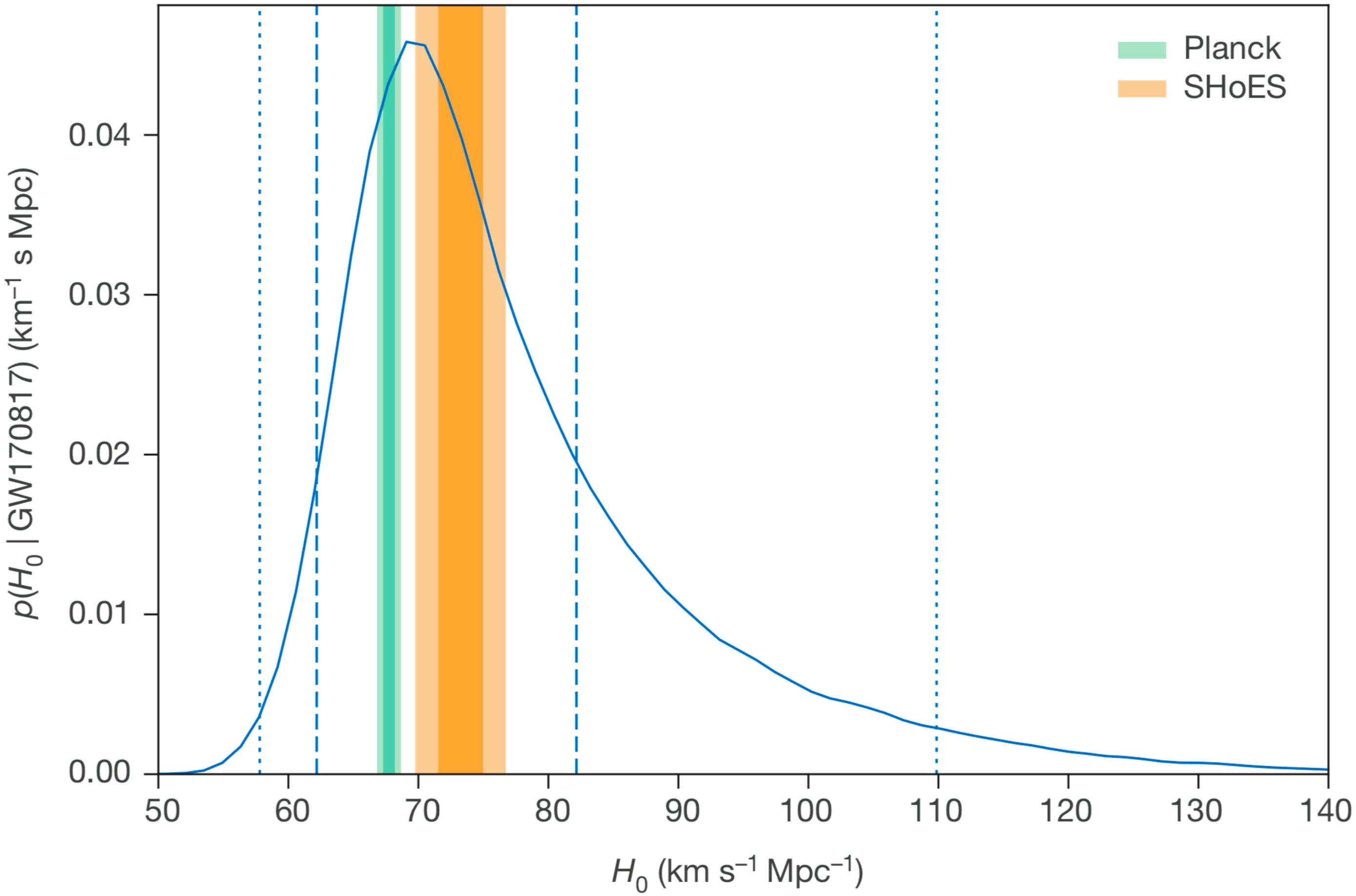
(Abbott et al. 2017)

H_0 from *one* BNS merger



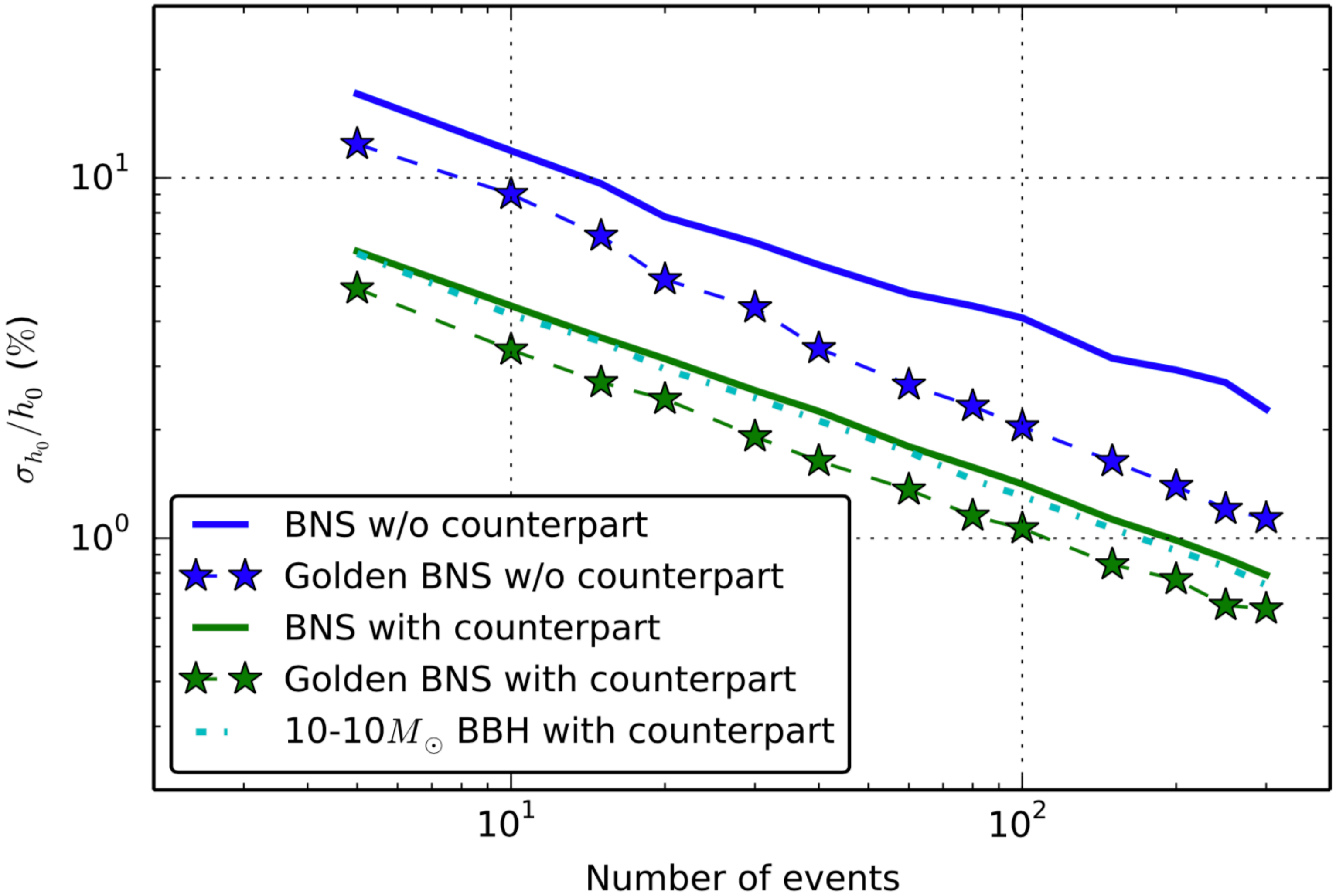
- GW 170817: LIGO+Virgo detected in GWs
- GRB 170807A: A GRB <2 seconds later
- Follow-up observations at all wavelengths
- Kilonova afterglow gives a location and host identification.
- GW data: "chirp mass" and distance D = 44(+7/-3) Mpc.
- Spectrum of host galaxy/group gives z = 0.0101+/-0.0006.
- Hubble constant estimate is $H_0 = c\ z\ /\ D$ = 70(+12/-8) km/s/Mpc.

H_0 from *one* BNS merger



(LIGO+Virgo 2017)

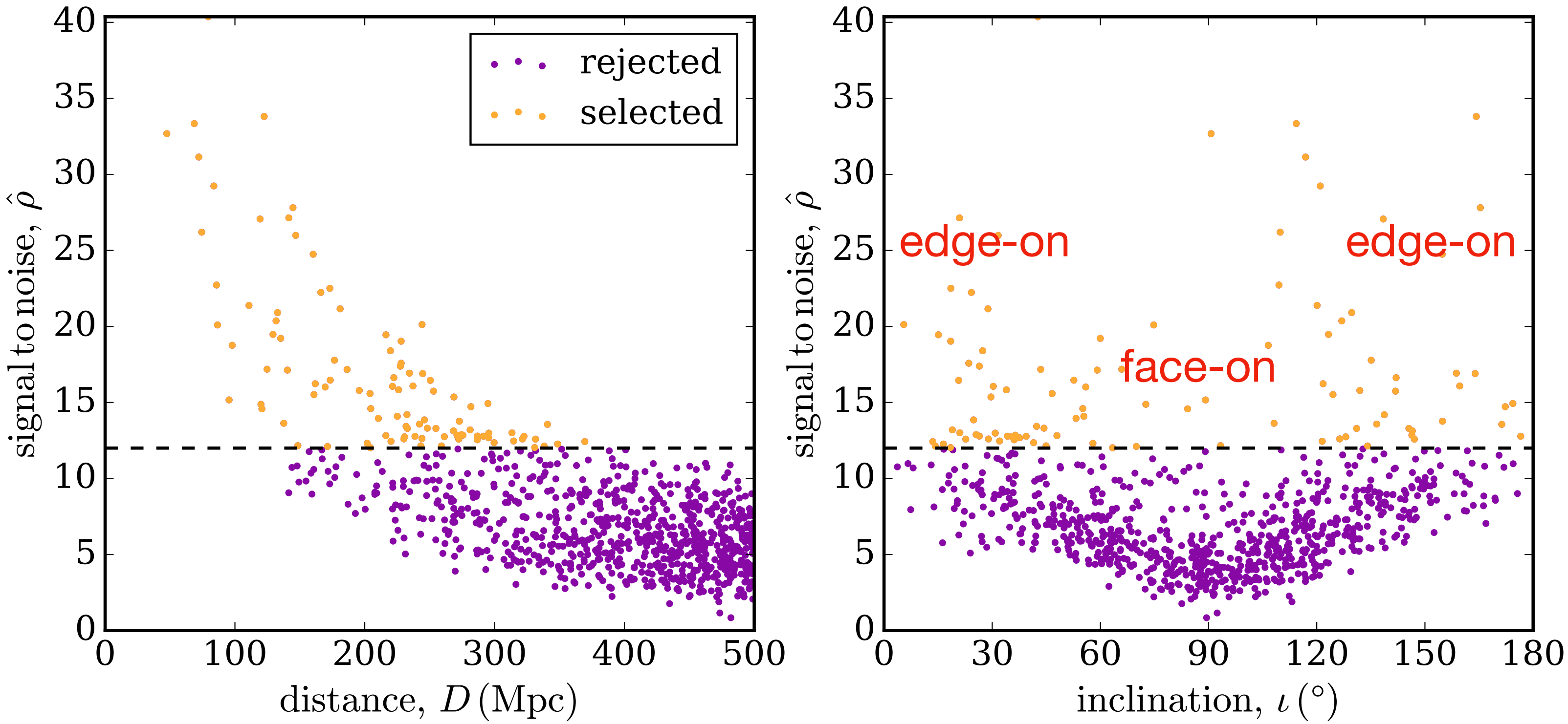
H_0 from *many* BNS mergers



1 km/s / Mpc

(Chen et al. 2018; *cf.* Dalal et al. 2006)

Simulations



Posterior distributions in H_0 from 25 simulations of samples of N BNSs

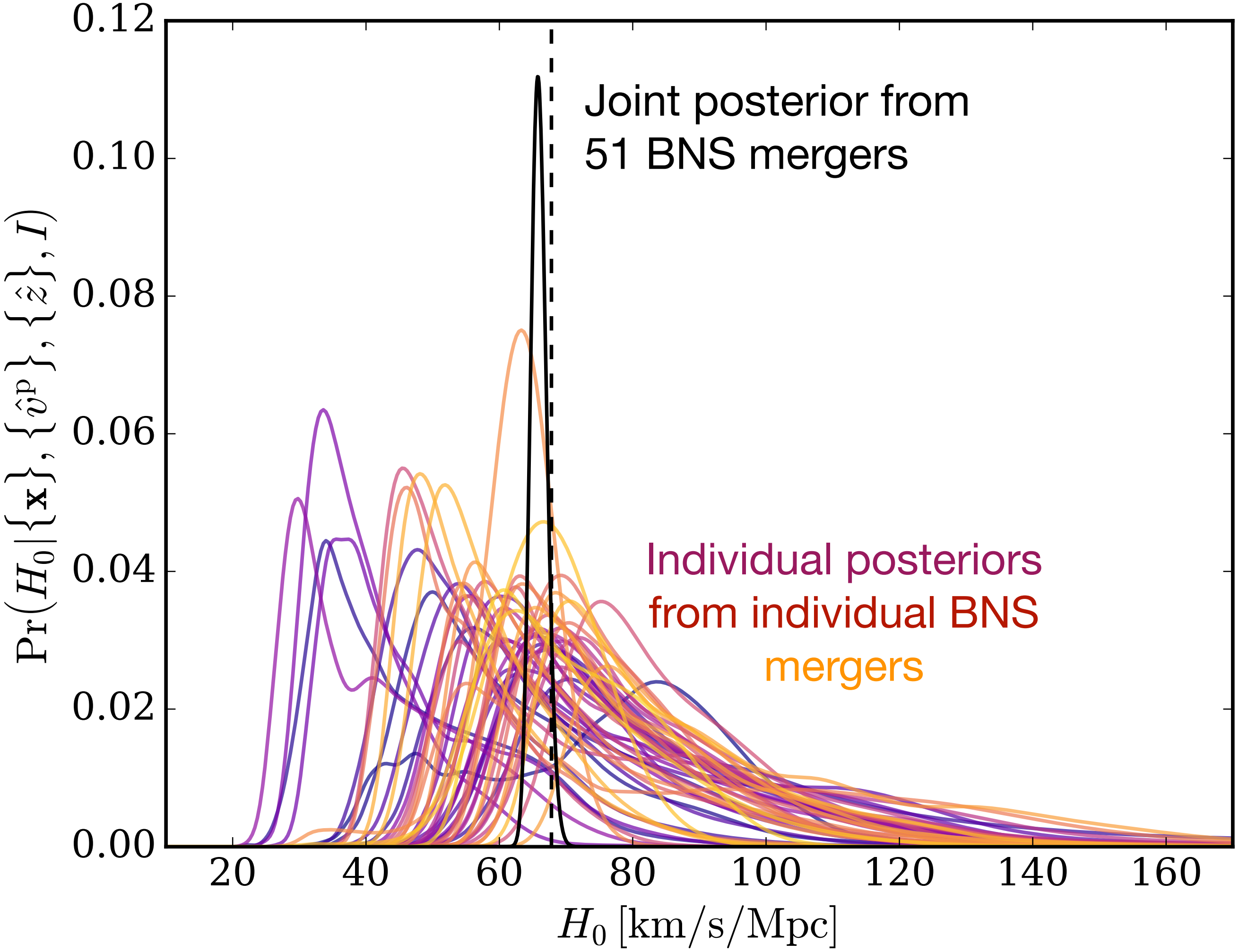
Full models too slow to do large number numbers of realisations.

Use linearised general relativity which includes only:
"chirp mass", M; distance, D; and inclination i.

Includes self-consistent selection on *observed* quantities.

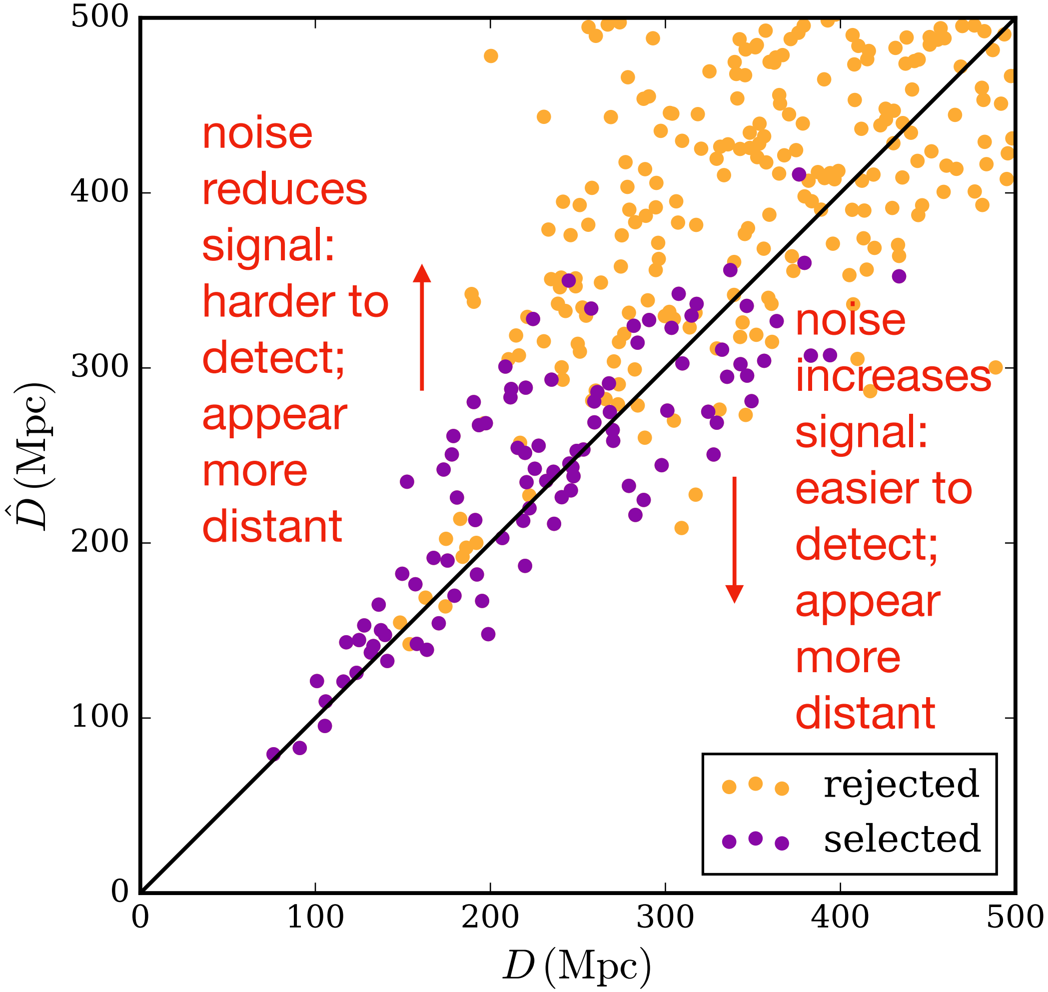
(Mortlock et al. 2019)

Simulation results



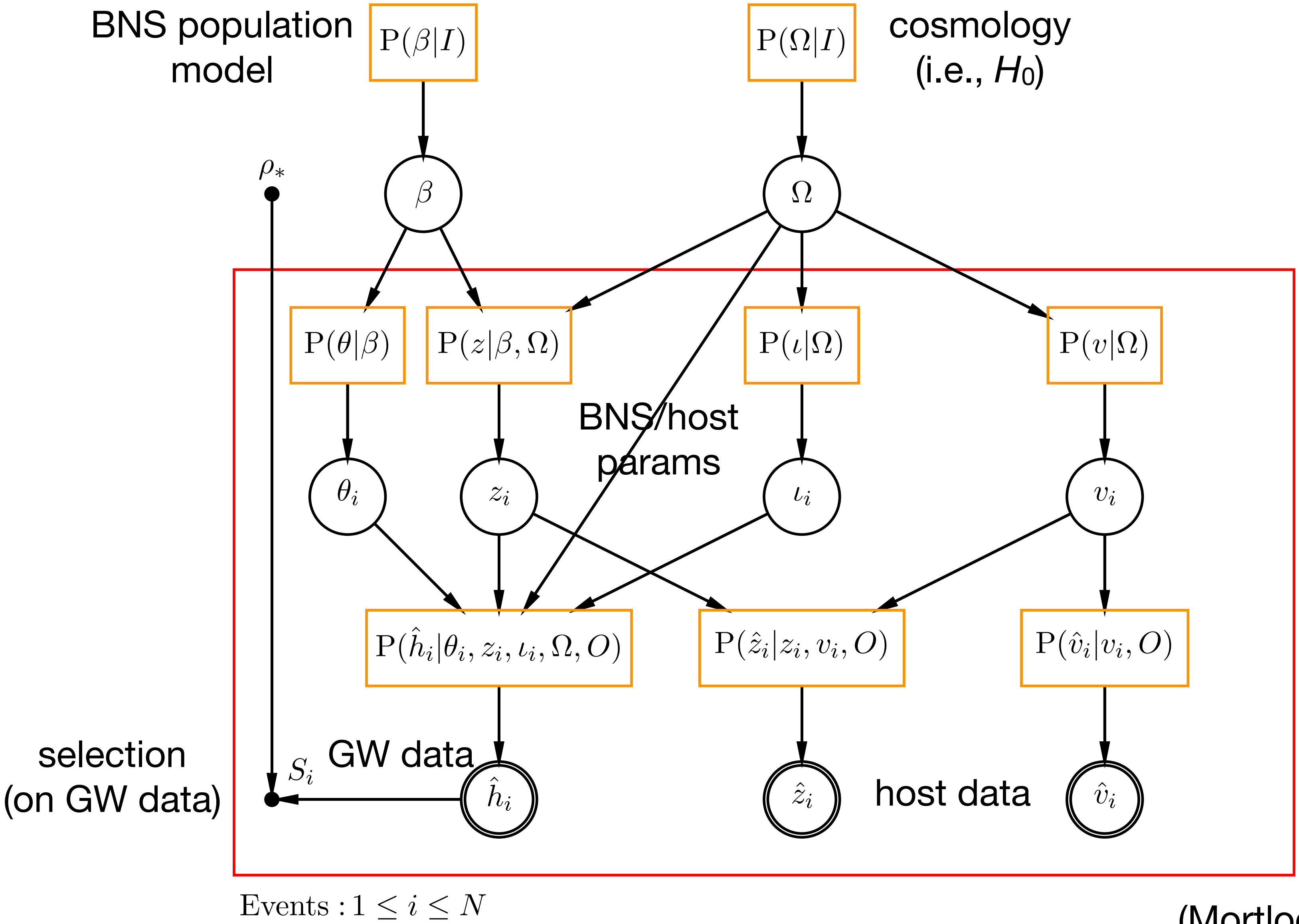
(Feeney et al. 2019)

Simulations



(Mortlock et al. 2019)

Statistical model



(Mortlock et al. 2019)

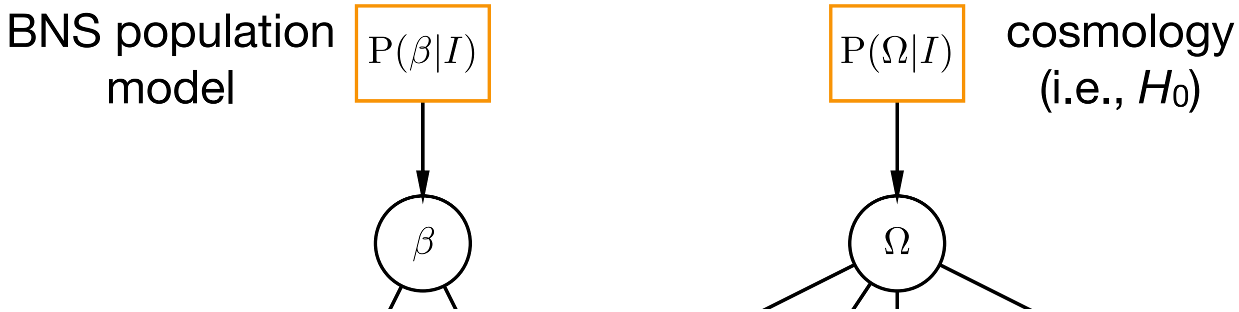
Statistical model

BNS population model $\mathrm{P}(\beta|I)$

$\mathrm{P}(\Omega|I)$ cosmology (i.e., H_0)

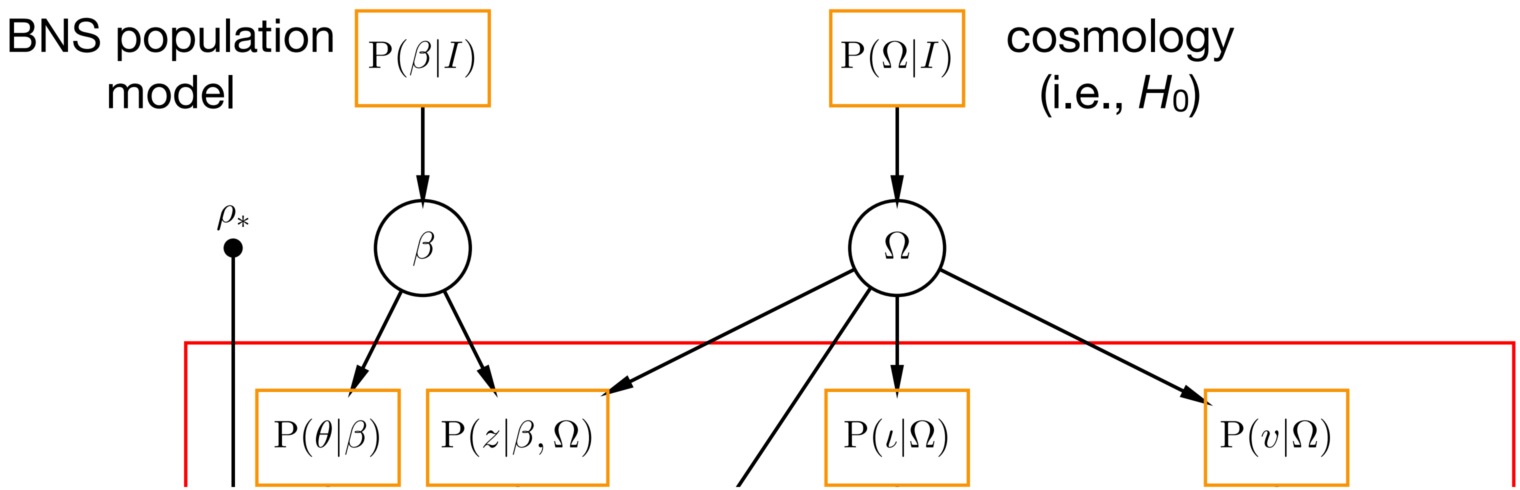
(Mortlock et al. 2019)

Statistical model



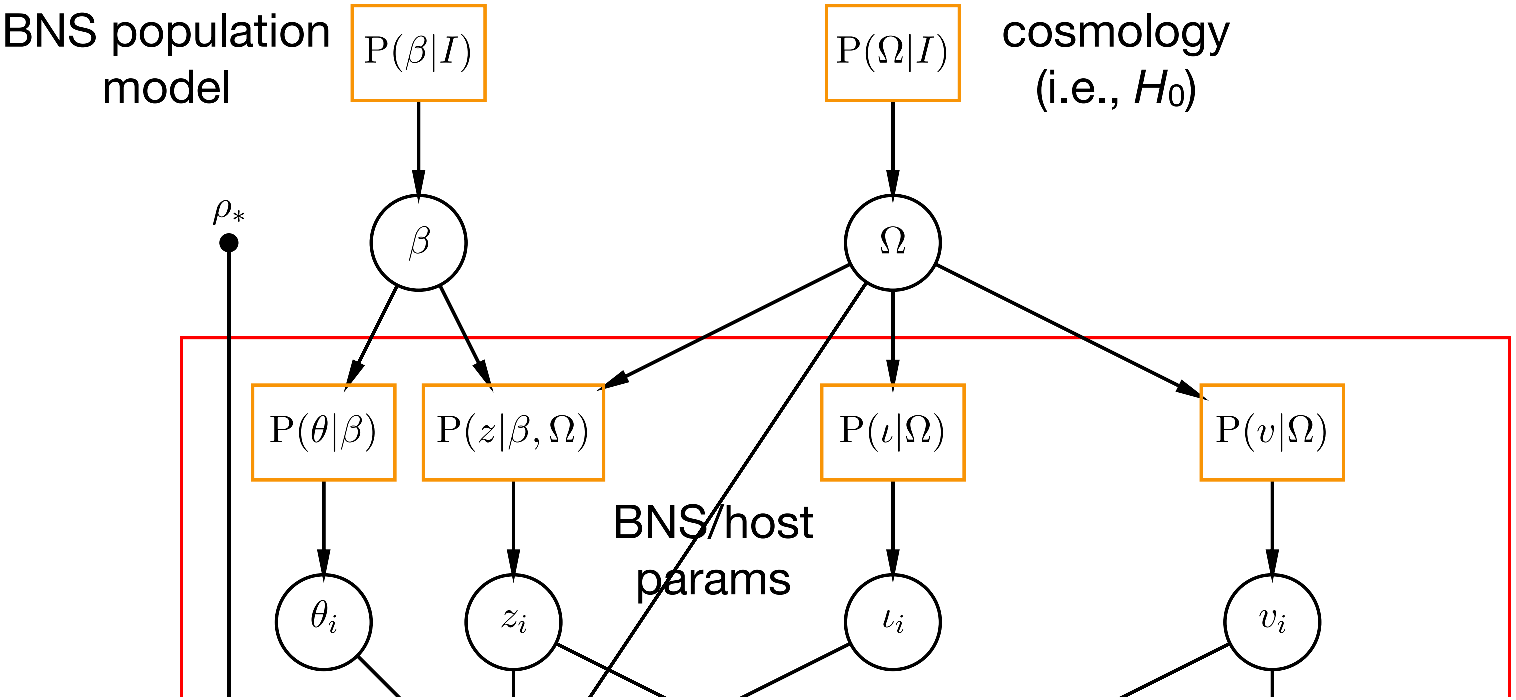
(Mortlock et al. 2019)

Statistical model



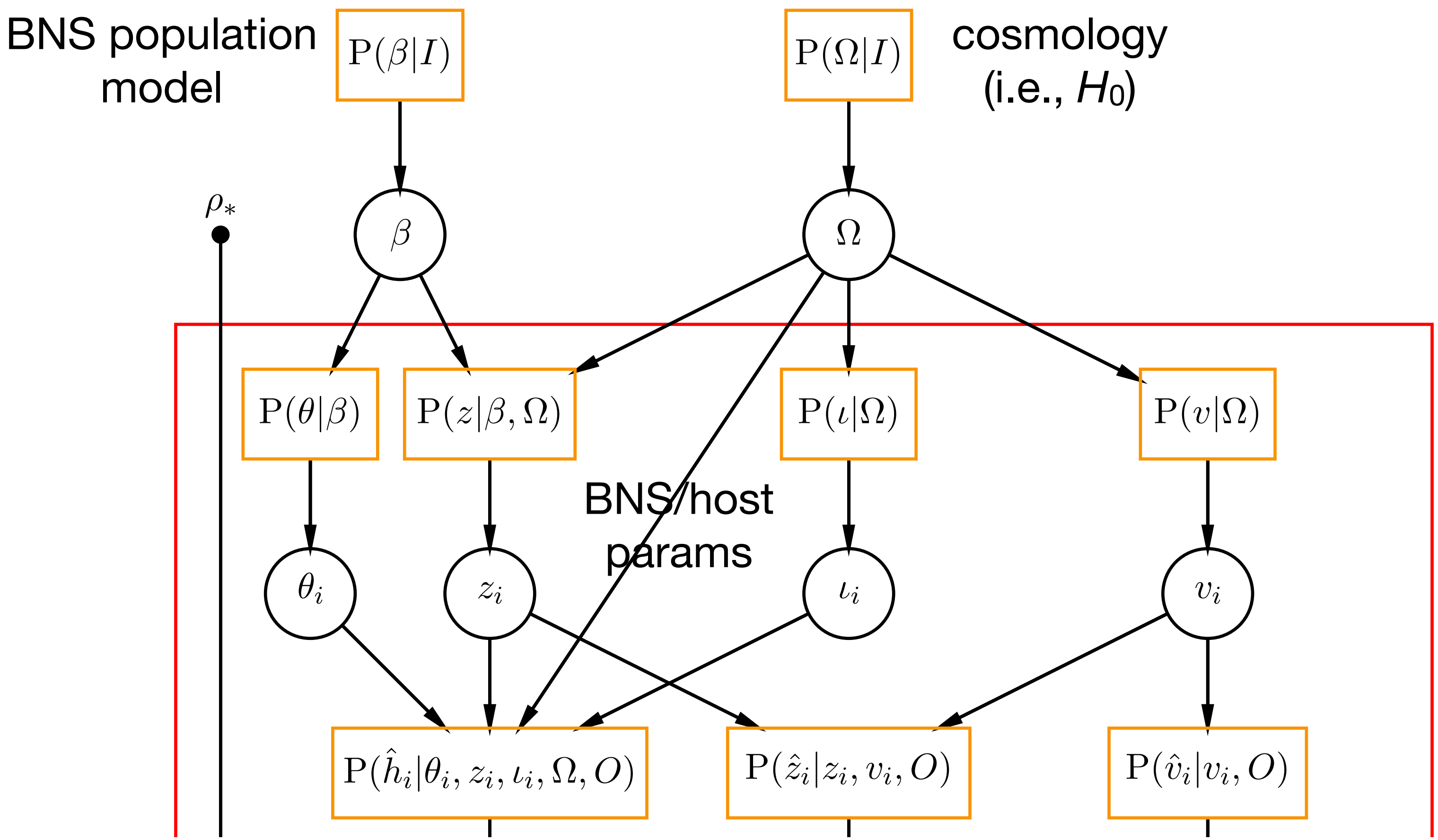
(Mortlock et al. 2019)

Statistical model



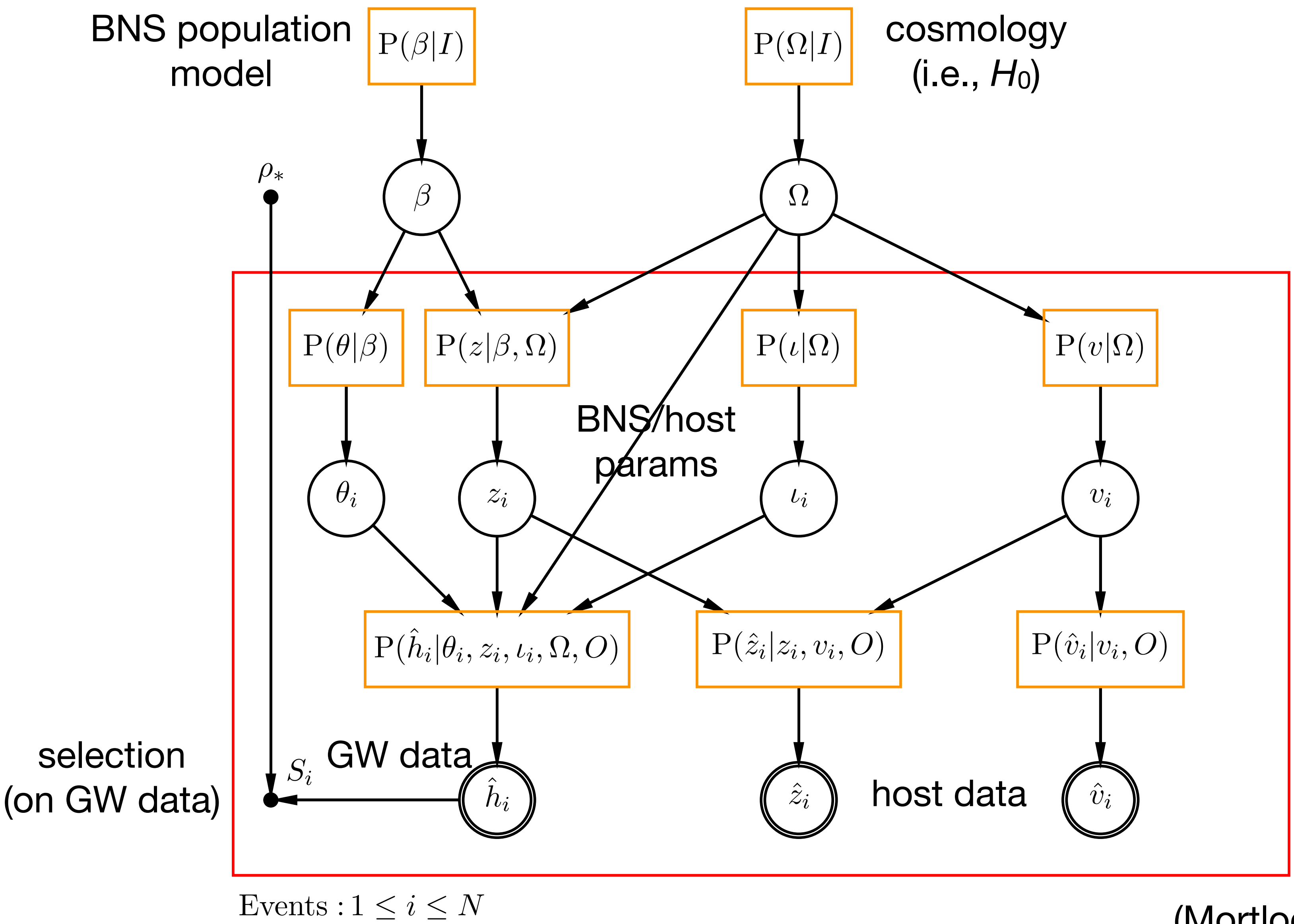
(Mortlock et al. 2019)

Statistical model



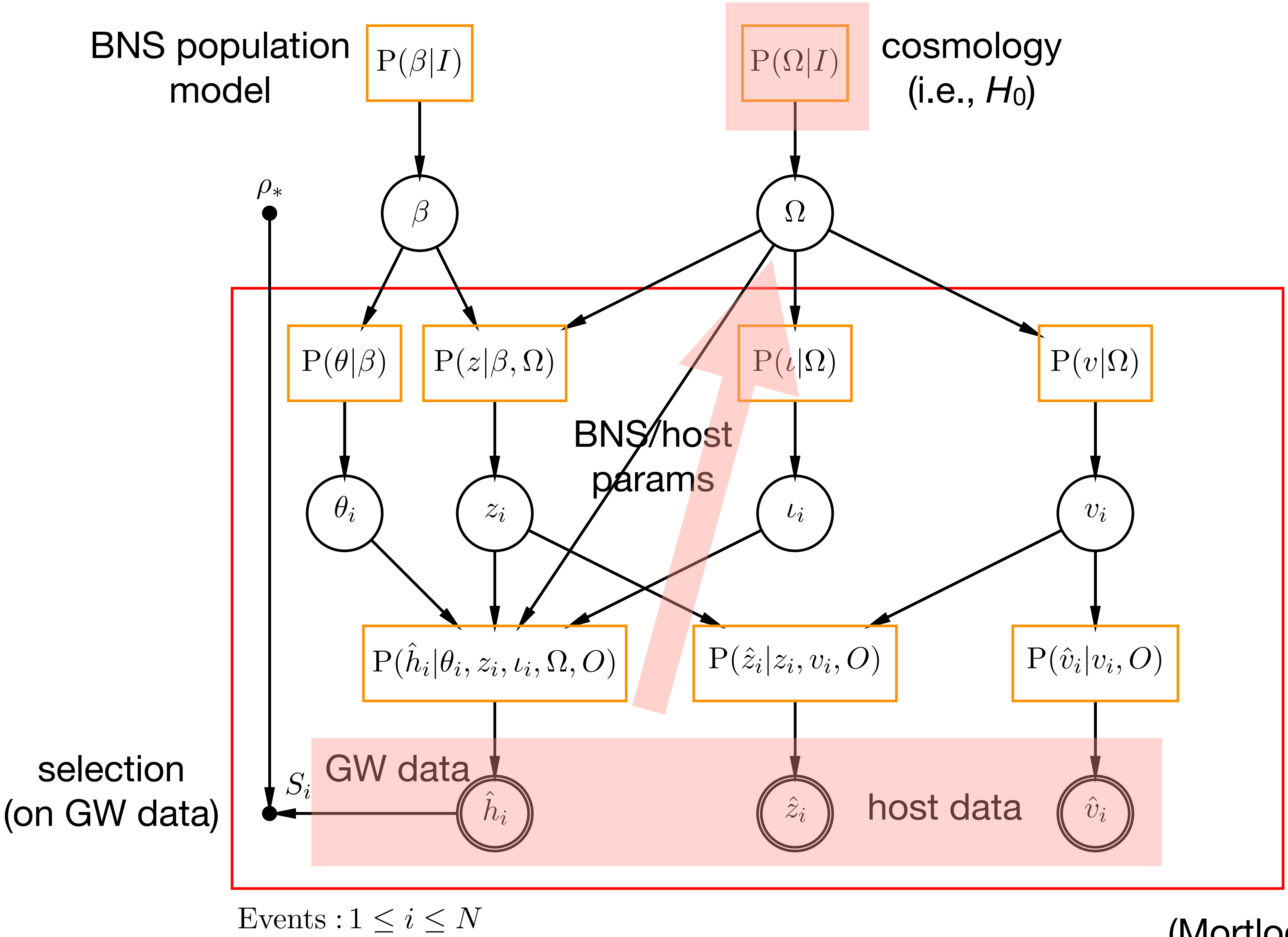
(Mortlock et al. 2019)

Statistical model



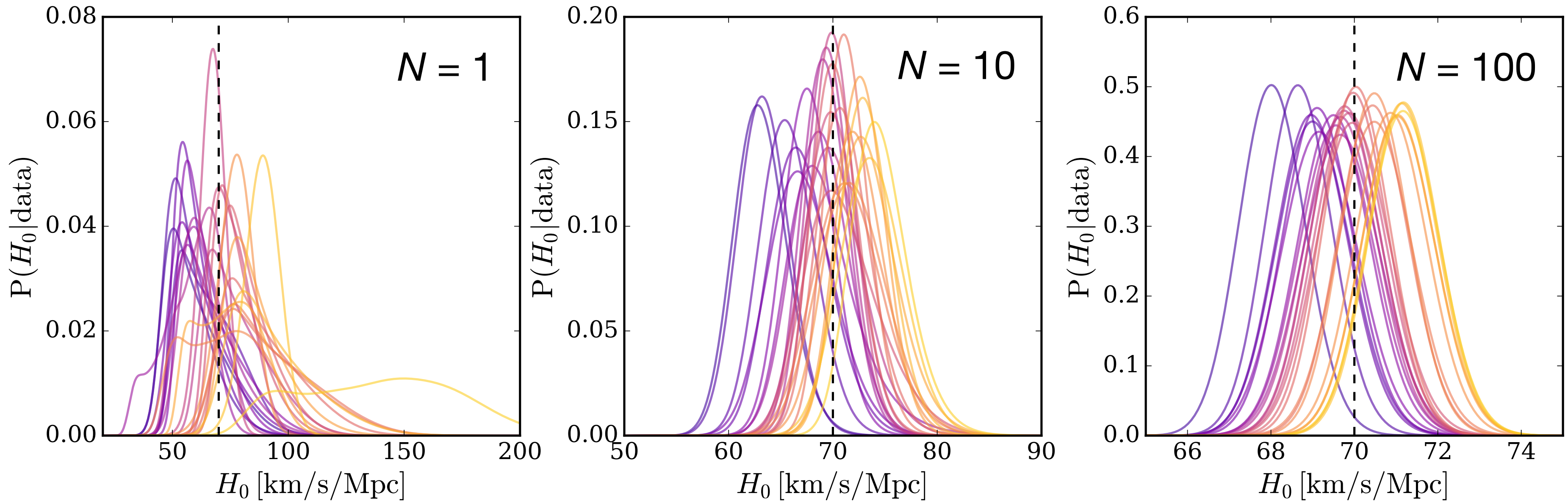
(Mortlock et al. 2019)

Statistical model



(Mortlock et al. 2019)

Simulation results



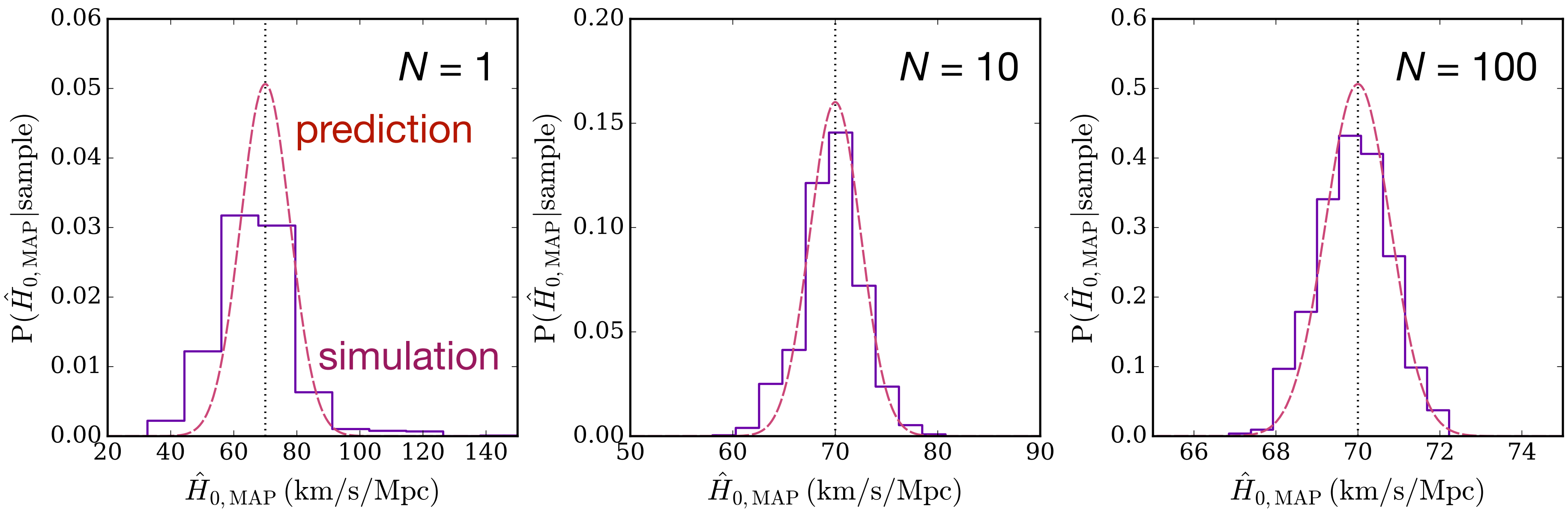
Posterior distributions in H_0 from 25 simulations of samples of N BNSs

N = 1: Posteriors have range of shapes; difficult to assess error/bias

N = 100: Posteriors all normal; into asymptotic regime

(Mortlock et al. 2019)

Removes potential bias



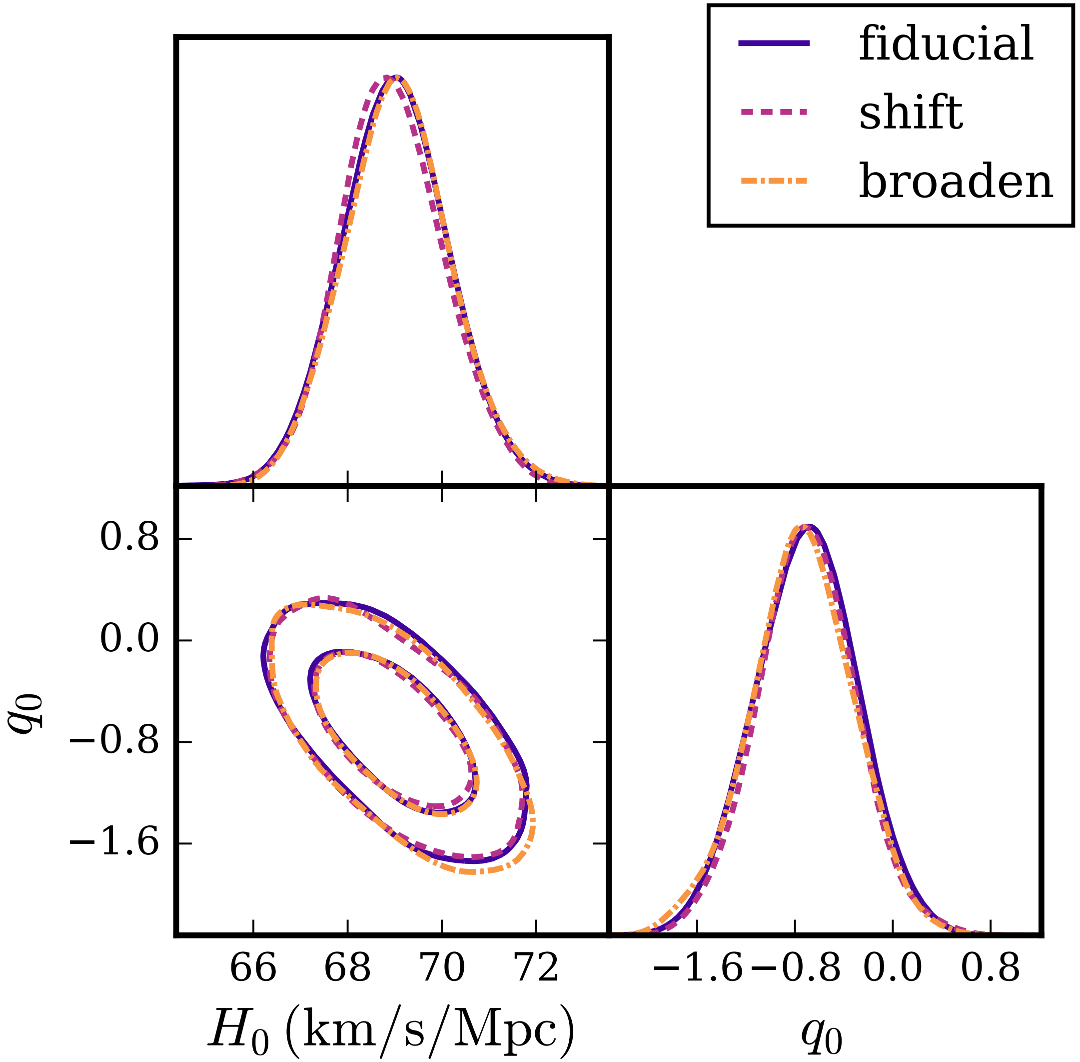
Distribution of MAP estimate of H_0 from simulations of samples of N BNSs

N = 1: Distribution has a high-H_0 tail; difficult to assess error/bias

N = 100: Distribution Gaussian; into asymptotic regime; unbiased (not guaranteed)

(Mortlock et al. 2018)

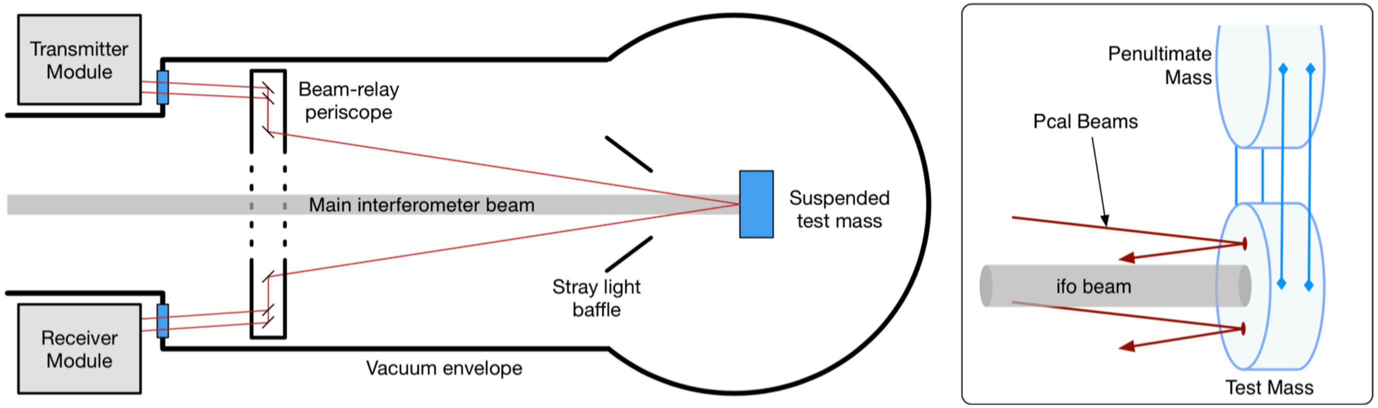
Robust to incorrect population models



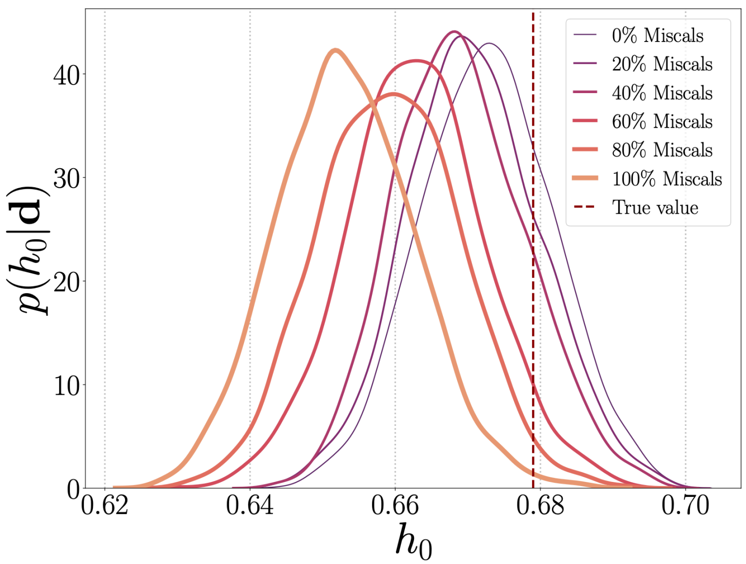
(Changes to mass distribution)

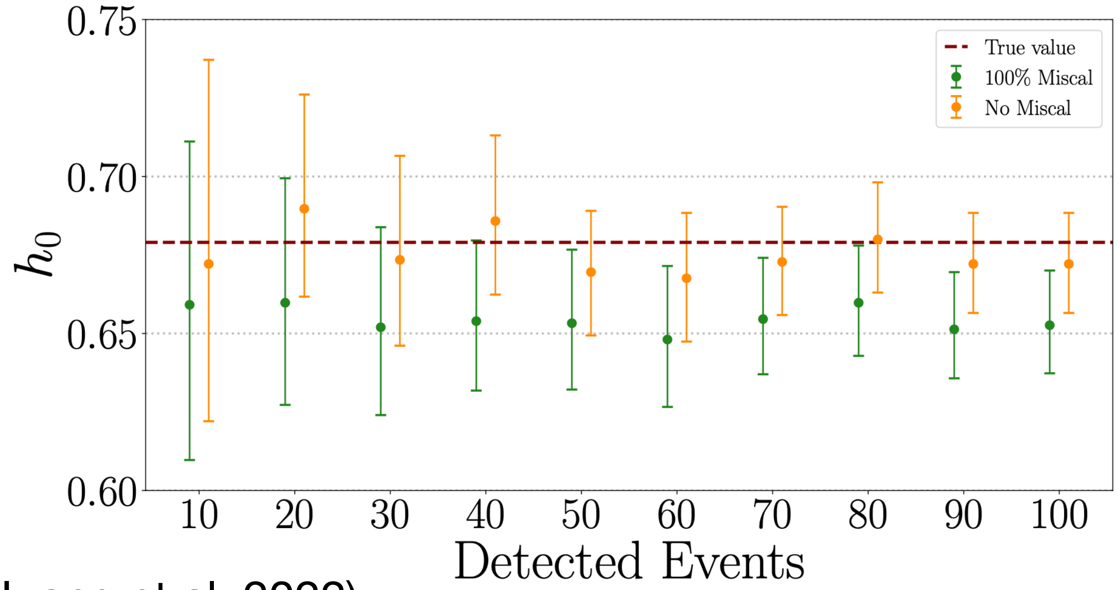
(Mortlock et al. 2019)

Absolute calibration



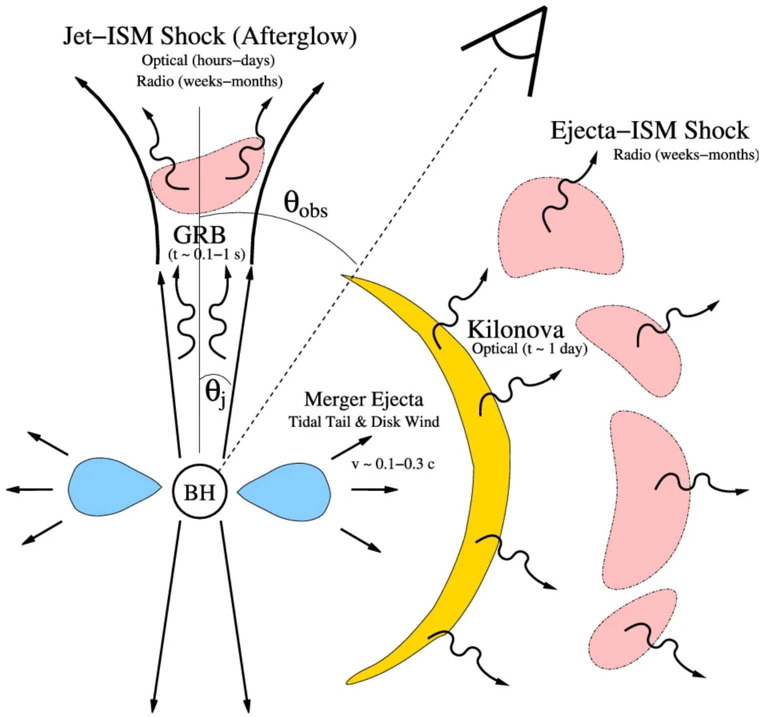
(Kakri et al. 2016; Callihane et al. 2017)



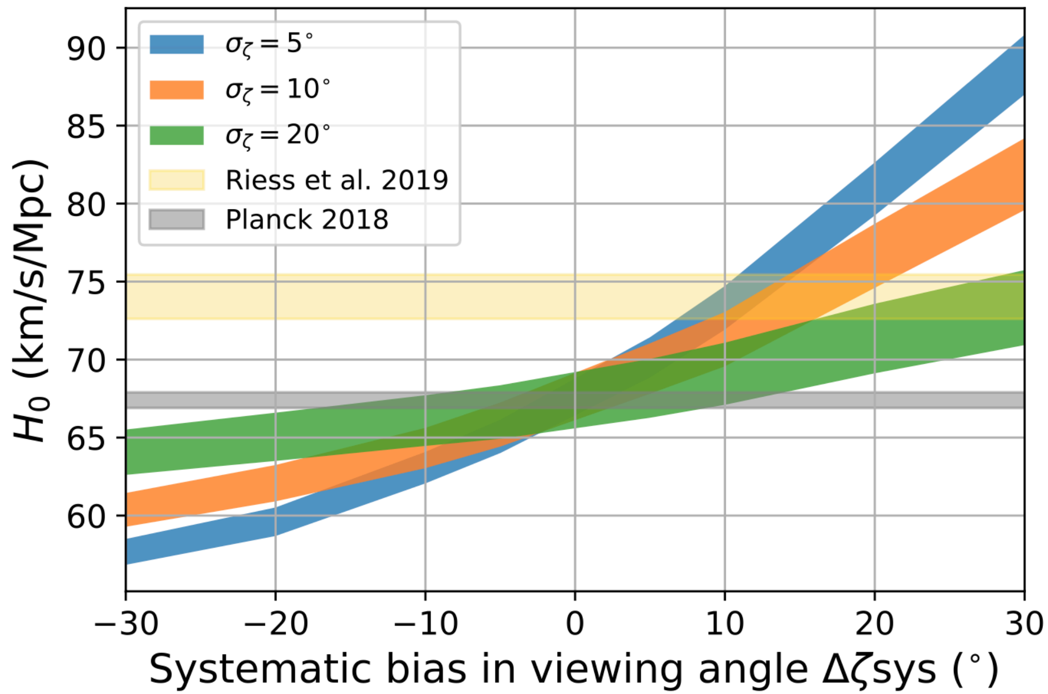


(Huang et al. 2022)

Optical/EM counterparts



(Metzger & Berger 2012)



(Chen 2020)

Conclusions

- Astronomical catalogues can be used to do a range of science.
- Possible to do object and population level analysis using Bayesian inference.
- Some level of approximation typically required but inference can be made robust to modelling assumptions.
- Critically, outputs are uncertainties and probabilities rather than estimates and classifications.