Going beyond isotropy and Gaussianity in Cosmology

Minkowski Functionals and other statistics

Javier **Carrón Duque**

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IAU-IAA Seminar

14 May 2024

I will talk about

- What are the main cosmological observables?
- Why are Gaussianity and isotropy important?
- What are Minkowski Functionals?
- Why are they useful?
- How can they be extended to CMB polarization?
- What about anisotropy?

Introduction

- What are the main cosmological observables?
- Why are Gaussianity and isotropy important?
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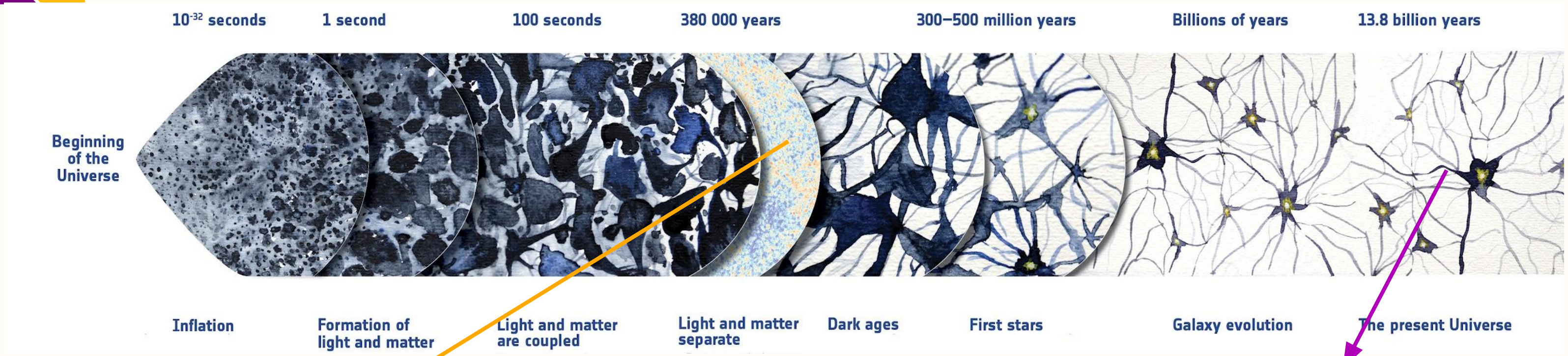
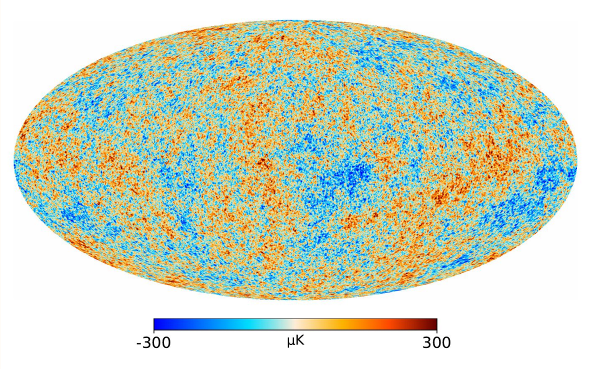
There are two main cosmological observables

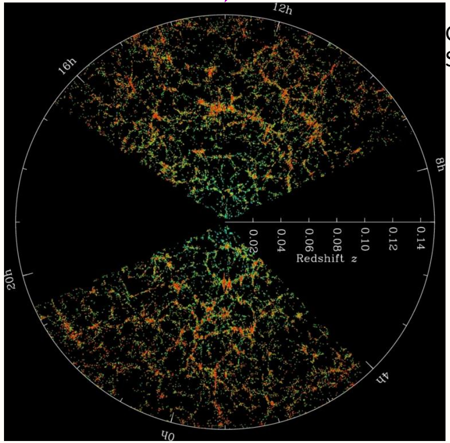


Credit: ESA



Planck data

← Cosmic Microwave Background (CMB)



Credit: SDSS

→ Large Scale Structure (LSS)

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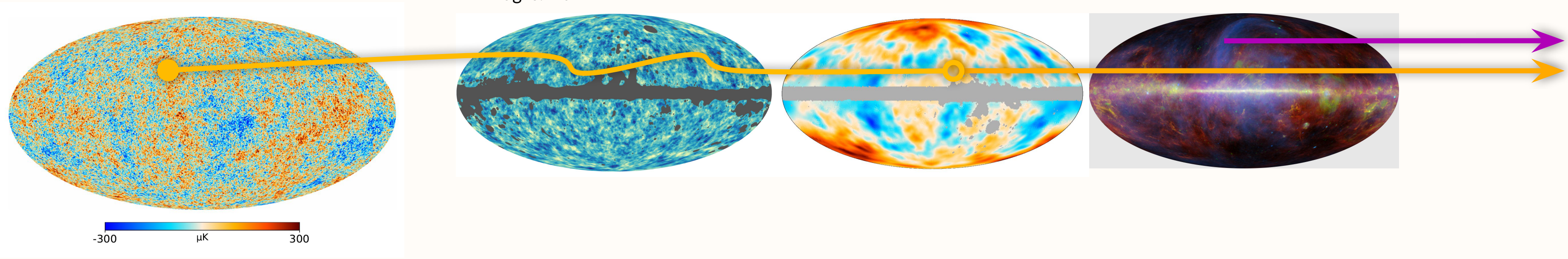
The Cosmic Microwave Background (CMB)

Planck data

Credit of the three images: Planck



Plasma becomes transparent
Photo of the plasma at this time
(~380 000 years)

Light is bent by the space curvature caused by the mass distribution

Light is modified by massive structures
(Integrated Sachs-Wolfe, Sunyaev-Zeldovich)

The galaxy emits light in similar wavelengths

(and more...)

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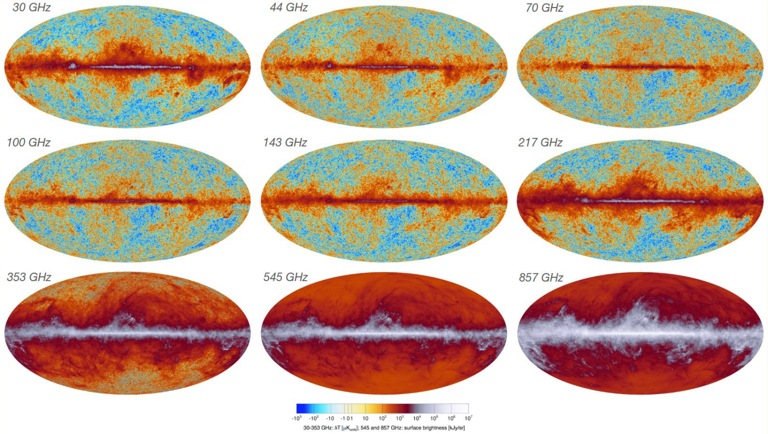
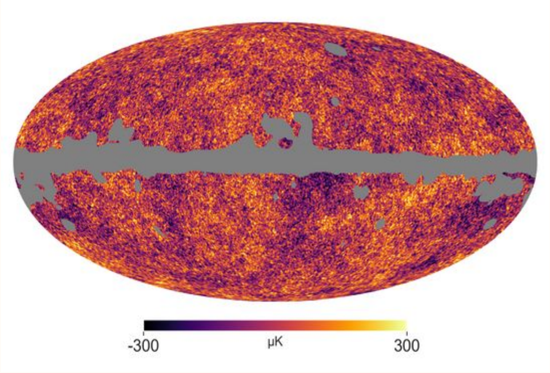
The Cosmic Microwave Background (CMB)

Credit: Planck



The CMB is observed in different wavelength bands



The CMB is reconstructed within the confidence mask through different methods

(Temperature and polarization information)

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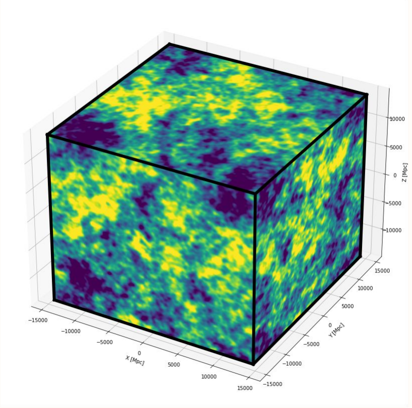
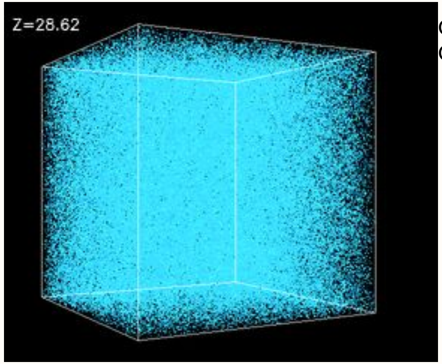
And the Large Scale Structure (LSS)

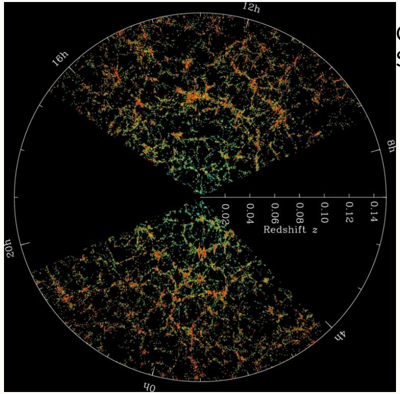


Initial conditions for the plasma
(Gaussian, homogeneous, isotropic)

Evolves due to gravity
and baryonic effects



Credit: Center for
Cosmological Physics



Credit:
SDSS

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Gaussian fields are simple

- Let $f(x)$ be a real random field
- We observe at points $\vec{x}_1, \ldots, \vec{x}_N$
- The field is ***Gaussian*** iff, for any set of observed points:
  - The probability distribution is a multivariate Gaussian $\mathcal{N}(\vec{\mu}, \Sigma)$

    With $\vec{\mu} = \{E[f(\vec{x}_1)], \ldots, E[f(\vec{x}_N)]\}$ the **expected value** at the points

    and $\Sigma_{ij} = \mathrm{Cov}[f(\vec{x}_i), f(\vec{x}_j)]$ the **covariance** between pairs
- Fully defined by 1-point and 2-point correlation functions!
- Higher order correlation functions are given by these two (Isserlis' Theorem)
- Primordial perturbations and CMB are Gaussian fields (or very close to it)

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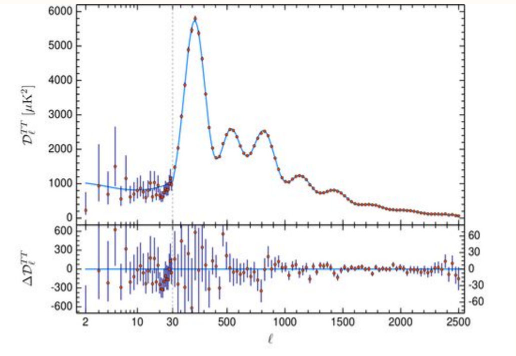
Homogeneity and isotropy are typically assumed

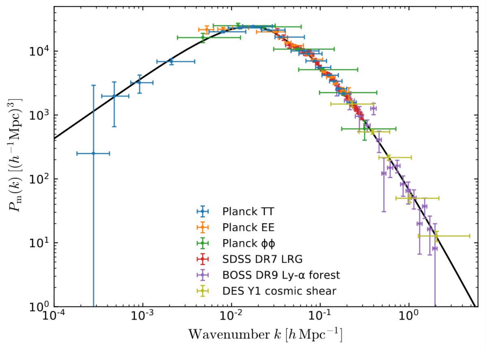
- Homogeneity: invariance to translations
  - The expected value is constant
  - The covariance only depends on the relative distance (vector)

- Isotropy: invariance to rotations
  - Typically requires a center
  - With homogeneity: the covariance only depends on the relative distance (modulus)

CMB Power spectrum



Matter power spectrum



Credit: Planck

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It is important to go beyond these assumptions

- (Near) Gaussianity in the primordial plasma is predicted by Inflation
- **Non–Gaussianity** can provide useful information
  - About Inflation and Early Universe Physics
  - (CMB) About Late Universe effects on the CMB
  - (LSS) About non–linear gravity and baryonic effects
  - About systematics, biases, foreground contamination, …

- Homogeneity and isotropy is assumed by the Cosmological Principle
- **Deviations** can provide useful information
  - About the topology of the Universe
  - About gradients in the Universe (*i.e.*, dipoles)
  - About the velocity of the Milky Way and other structures
  - About systematics, biases, foreground contamination, …

Other statistics can probe further information

- *N* points correlation functions (bispectrum, trispectrum, ...)
- Anisotropic power spectrum, density-split power spectrum, ...
- Extrema statistics (maxima, minima, saddle points)
- Stacking
- Topological descriptors: **Minkowski Functionals**, Betti numbers, persistent homology
- Wavelet Scattering transform
- Field-level inference
- Machine Learning

- f_{NL} is very important, but not the only way

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Minkowski Functionals

- What are the main cosmological observables?
- Why are Gaussianity and isotropy important?
- What are Minkowski Functionals?
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- How can they be extended to CMB polarization?
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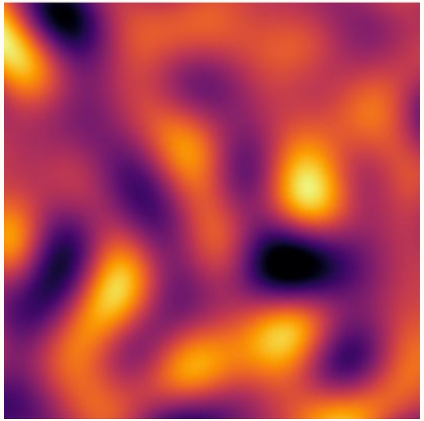
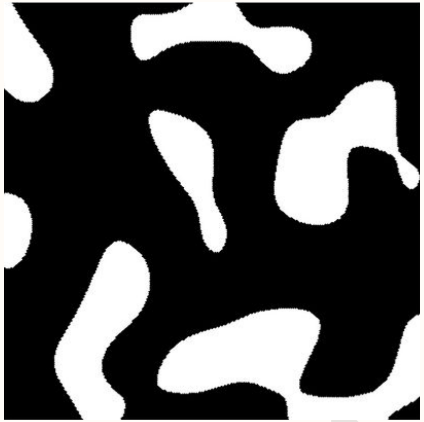
Minkowski Functionals describe the geometry and topology of the field





- We consider a field (*e.g.*, T or δ)
- Let u be a threshold (*e.g.*, 2σ)
- We define the **excursion set** $A(u)$ as the regions of the field above u
- **Minkowski Functionals** (MFs) are:
  - V_0 : area of $A(u)$
  - V_1 : boundary length of $A(u)$
  - V_2 : Euler—Poincaré characteristic of $A(u)$ (#regions – #holes)
- They are higher–order statistics (complement N-point correlation functions)

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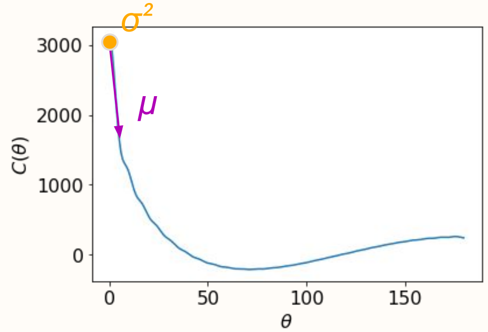
MFs are accurately predicted

- For Gaussian isotropic fields, the expected value is known and the variance is small
- It is **independently** affected by only three factors: threshold, manifold, correlation length

$$\mathbb{E}[V_j(A_u)] \approx \underbrace{\rho_j(u)}_{\text{Threshold}} \underbrace{V_0(\mathbb{S}^2)}_{\text{Ambient manifold}} \underbrace{\mu^{j/2}}_{\text{Correlation length of the map}}$$

- $\rho_j(u)$: gaussian × Hermite polynomial
- $V_0(\mathbb{S}^2) = 4\pi \times [f_{sky}]$
- μ: inverse of correlation length



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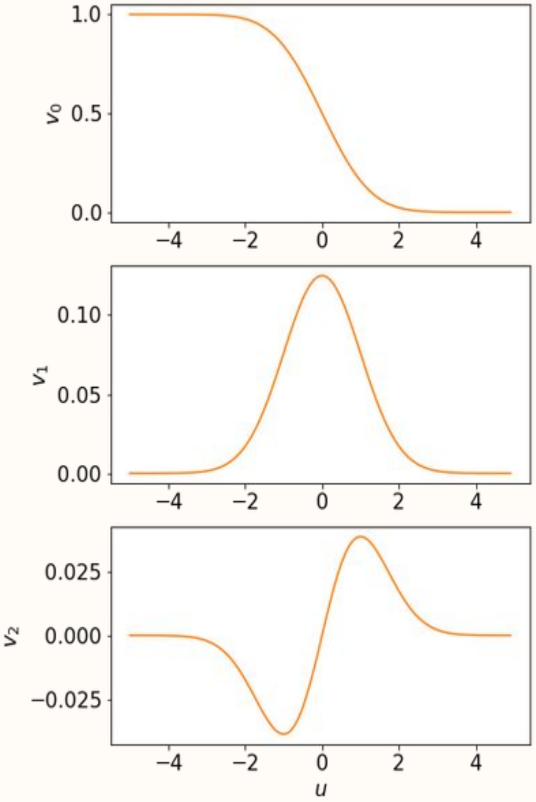
MFs are accurately predicted



$$\frac{\mathbb{E}\left[V_0(A_u)\right]}{4\pi} = 1 - \Phi(u)$$
$$\frac{\mathbb{E}\left[V_1(A_u)\right]}{4\pi} = \frac{1}{8}\exp(-\frac{u^2}{2})\mu^{1/2}$$
$$\frac{\mathbb{E}\left[V_2(A_u)\right]}{4\pi} = \frac{\mu}{\sqrt{(2\pi)^3}}\exp(-\frac{u^2}{2})u$$

$$P(f, f_x, f_y, f_{xx}, f_{xy}, f_{yy})$$

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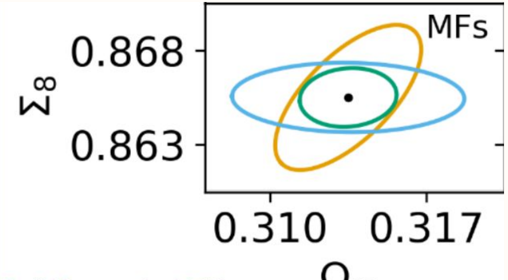
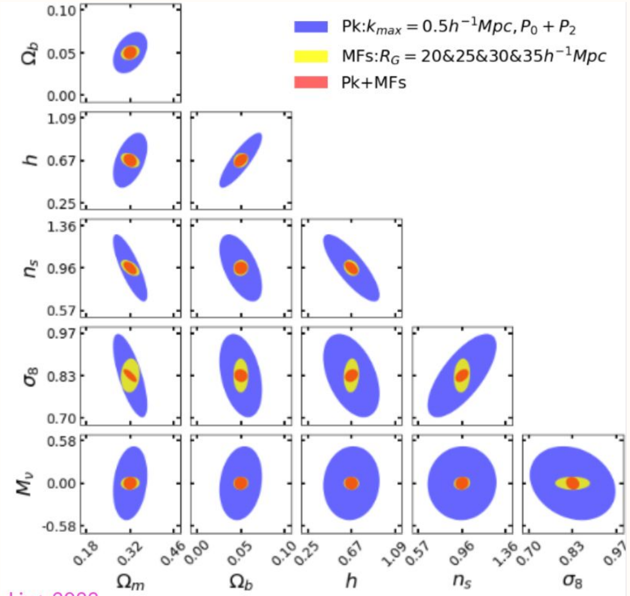
MFs have plenty of applications



Euclid Preparation XXIX

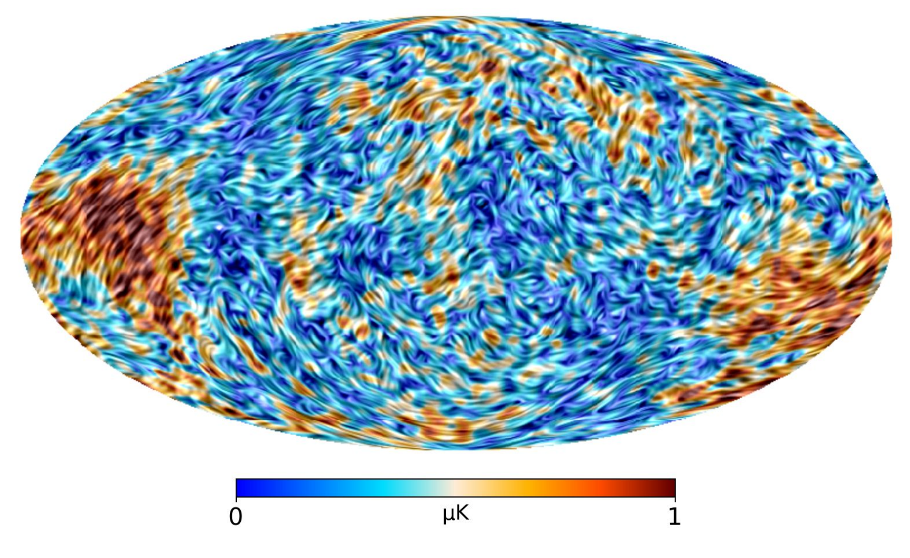
- Any deviation is due to non—Gaussianity and/or anisotropy
- Early Universe (*e.g.*, T): test for primordial non—Gaussianity
  - Planck 2018 VII (isotropy & statistics)
- Late Universe (*e.g.*, κ): extract more cosmological information
  - Euclid Preparation XXIX (2023), Grewal+ (2022), Zürcher+ (2022), …
- Foregrounds (*e.g.*, Galactic):
  - Martire+ (2023), Krachmalnicoff+ (2020), …
- Large Scale Structure (*e.g.*, galaxy distribution):
  - Liu+ (2023), Jiang+ (2023) Appleby+ (2022), Spina (2021), …



Liu+ 2023

We want to extend MFs to CMB polarization

- CMB polarization is an information-rich complement to CMB temperature
- It is a **complex** spin-2 field defined on the sphere
- We extend the MF formalism in two ways



We extend MFs to the modulus of polarization P^2

arXiv: 2211.07562

Minkowski Functionals of CMB polarization intensity with Pynkowski: theory and application to *Planck* and future data

Alessandro Carones[1,2★], Javier Carrón Duque[1,2], Domenico Marinucci[3], Marina Migliaccio[1,2], Nicola Vittorio[1,2]

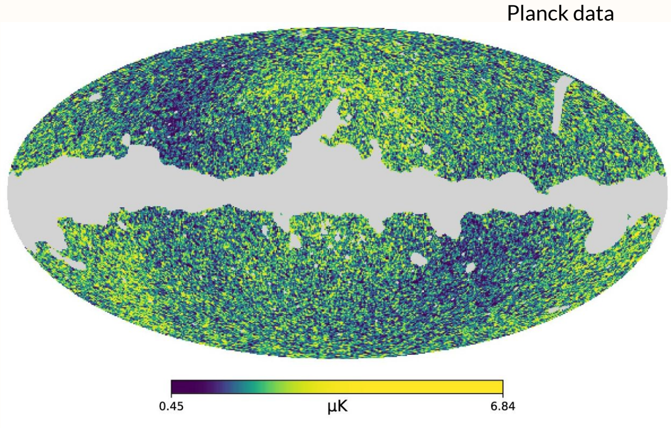
- We generalize the theoretical formula for $P^2 = Q^2 + U^2$
- This field is not Gaussian (χ^2 + spin effects)

$$\frac{\mathbb{E}\left[V_0(A_u)\right]}{4\pi} = \exp\left(-\frac{u}{2}\right)$$

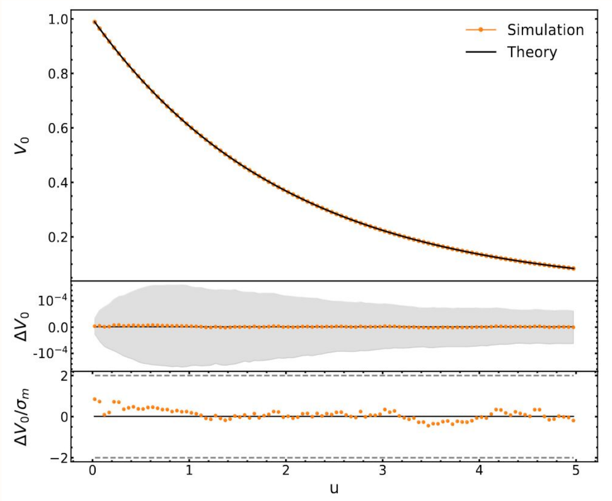
$$\frac{\mathbb{E}\left[V_1(A_u)\right]}{4\pi} = \frac{\sqrt{2\pi}}{8}\sqrt{\mu u}\exp\left(-\frac{u}{2}\right)$$

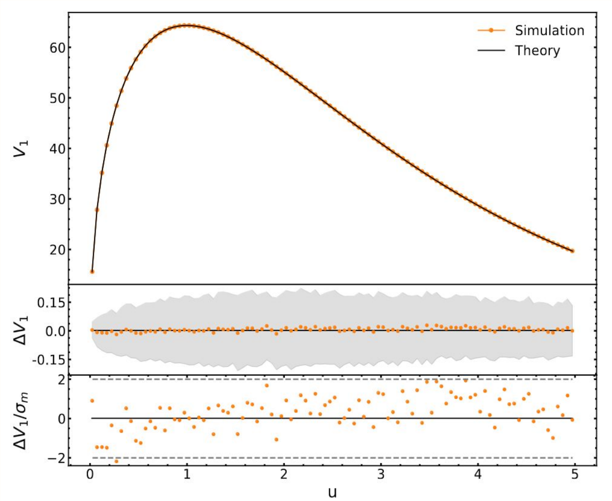
$$\frac{\mathbb{E}\left[V_2(A_u)\right]}{4\pi} = \mu\frac{(u-1)\exp(-u/2)}{2\pi}$$

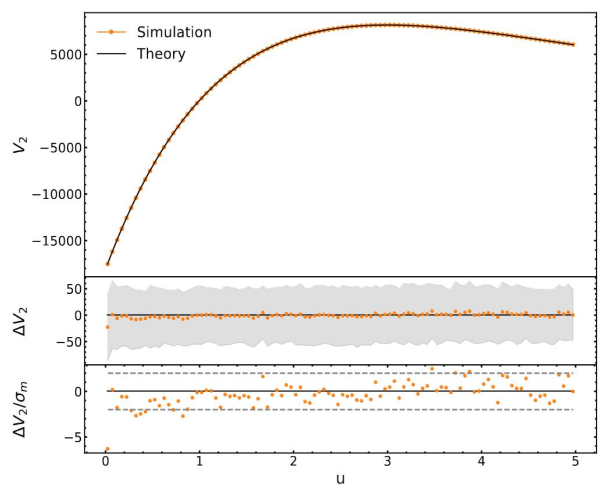
We verify theory and simulations



- Simulation are fully compatible with predictions (shown below with residuals and residual over std)
- But realistic simulations must include anisotropic noise (observational strategy)







arXiv: 2211.07562

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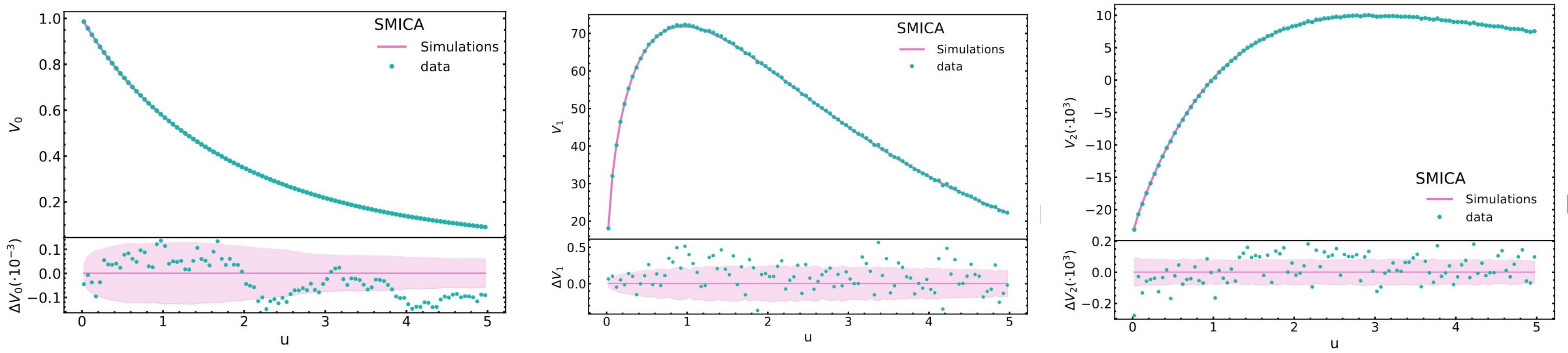
Planck P^2 is compatible with realistic simulations

- Realistic simulations must include anisotropic noise
- There is no significant deviation (SMICA & SEVEM)
- Significant improvement for **future observations**

		χ^2	p_{exc} (%)
V_0	SMICA	1.02	43.7
	SEVEM	0.81	75.3
V_1	SMICA	0.91	60.7
	SEVEM	1.36	8.3
V_2	SMICA	1.22	21.0
	SEVEM	0.82	68.0



arXiv: 2211.07562

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There is more information in the polarization field

- Polarization is a spin–2 complex field
- Information is lost in any scalar projection (P, E, B, Q, U, ...)
- We analyze the full polarization information using

$$f(\phi, \theta, \psi) = Q(\phi, \theta)\cos(2\psi) - U(\phi, \theta)\sin(2\psi)$$

- This is defined in *SO(3)*, a 3D manifold

arXiv: 2301.13191

Minkowski Functionals in $SO(3)$ for the spin–2 CMB polarisation field

J. Carrón Duque,[a,b,1] A. Carones,[a,b] D. Marinucci,[c] M. Migliaccio,[a,b] and N. Vittorio[a,b]

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We obtain the theoretical expectation

- More complicated case due to:
  - Anisotropy of the field (in the ψ direction)
  - Non–diagonal metric
- Expanding the formalism, we obtain:

$$\mathbb{E}\left[v_0\right] = 1 - \Phi(u)$$

$$\mathbb{E}\left[v_1\right] = K_1 \frac{5}{6\pi\sqrt{2}} \mu^{1/2} \exp\left(\frac{-u^2}{2}\right)$$

$$\mathbb{E}\left[v_2\right] = K_2 \frac{5}{24\pi^{3/2}} \mu \, u \exp\left(\frac{-u^2}{2}\right)$$

$$\mathbb{E}\left[v_3\right] = \frac{5}{8\sqrt{2}\,\pi^2} \mu \left(u^2 - 1\right) \exp\left(\frac{-u^2}{2}\right)$$

arXiv: 2301.13191

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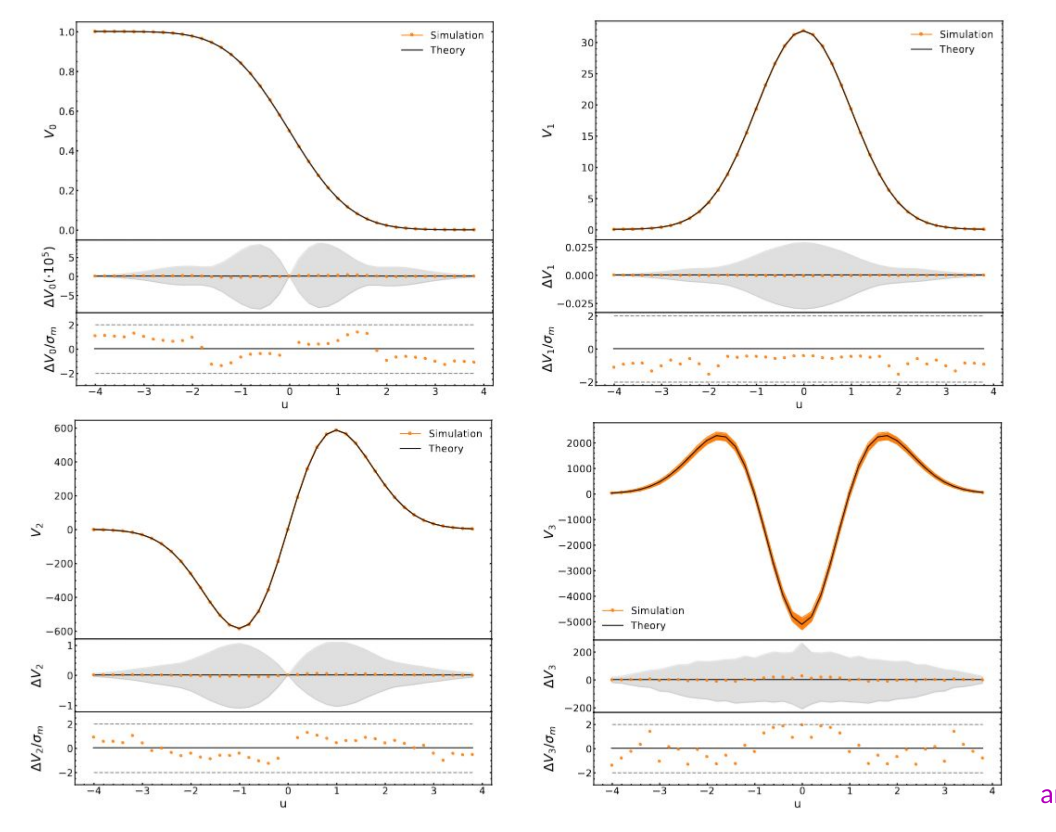
Simulations are fully compatible with theory



- 4 MFs in 3D space
- No significant deviation found
- New tool to explore polarization:
  - Blind deviations
  - Cleaning techniques
  - Foreground models
  - Non–Gaussianity/ anisotropy

arXiv: 2301.13191

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MFs can be applied to the 3D density field

- The LSS is NOT Gaussian: lots of information in its non—Gaussianities

Primordial non—Gaussianities

- Consequence of Inflation
- MFs are well suited for some models
- Blind or model dependent

Late Universe non—Gaussianities

- Consequence of Gravity and Baryonic effects
- Dominant, especially at small scales

- Can MFs distinguish both origins?
  - Can we include the effect of Gravity?
- Can MFs constrain cosmological parameters?
  - Yes, at least with forward modelling
- How do they compare to other statistics?
  - Theoretical models, degeneracies, systematics, ...

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MFs can have many other applications

- We are looking into:
    - Galactic dust polarized emission
    - Morphology of the LSS
    - Forecasts for future missions
    - CMB power asymmetry
    - **+ new ideas?**

We developed Pynkowski as a Python package

- Pynkowski is **fully documented** and modular
  - Theory module: theoretical predictions of different kinds of fields (Gaussian, χ^2, f, ...)
  - Data module: different kinds of data structures (numpy arrays, healpix maps, ...)
  - Stats module: different higher–order statistics (MFs, maxima/minima distribution, ...)
- All modules are easy to use and expand

Now available!

https://github.com/javicarron/pynkowski

```
$ pip install pynkowski
```

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Pynkowski is easy to use

https://github.com/javicarron/pynkowski

```
$ pip install pynkowski
```

```python
import numpy as np
import healpy as hp

import pynkowski as mf    # For Minkowski Functionals

# Define the thesholds for the excursion sets
us = np.linspace(-5., 5., 100)

# Load the CMB map and angular power spectrum
my_map = ...
my_cls = hp.anafast(my_map)    # or load from file

# Compute the Minkowski Functionals on my map
data_map = mf.Healpix(my_map, normalise=True, mask=None)     # Default parameters

v0_data = mf.V0(data_map, us)
v1_data = mf.V1(data_map, us)
v2_data = mf.V2(data_map, us)

# Compute the Minkowski Functionals on a Gaussian random field with the same power spectrum
gaussian_field = mf.SphericalGaussian(my_cls, normalise=True, fsky=1.)       # Default parameters

v0_theory = mf.V0(gaussian_field, us)
v1_theory = mf.V1(gaussian_field, us)
v2_theory = mf.V2(gaussian_field, us)
```

Python

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Beyond isotropy

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Anisotropy: how and why?

- Homogeneity and isotropy simplify the 1 and 2-point correlation functions:

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = \begin{cases} C_\ell \, \delta^K_{\ell,\ell'} \delta^K_{m,m'} \\ C_{\ell m \ell' m'} \end{cases} \qquad \langle \delta(\vec{k}) \delta^*(\vec{k}') \rangle \propto \begin{cases} P(|\vec{k}|) \, \delta^D(\vec{k} - \vec{k}') \\ P(\vec{k}, \vec{k}') \end{cases}$$

- Hints for possible violation of statistical isotropy at large scales
- CMB anomalies: Lack of correlations at $\theta > 60\,\mathrm{deg}$, power asymmetry, quadrupole–octupole alignment, parity violation
- Kinematic dipole: tension between CMB and LSS
- Bulk flows at large scales

Some models predict anisotropy

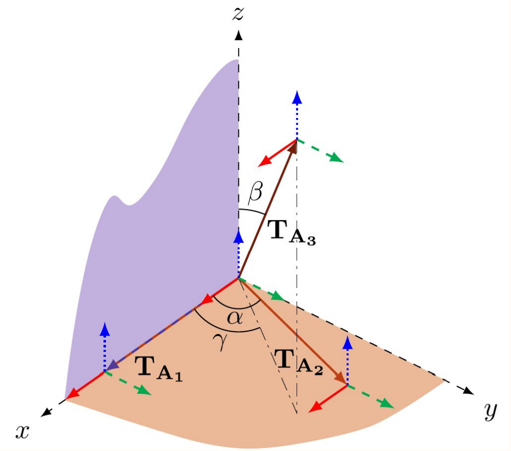
- There are phenomenological and theoretical models that break isotropy and homogeneity
- Tilted cosmology, modulating fields, …
- Bianchi Universe
- **Non–trivial topology**

- FLRW metric (locally homogeneous and isotropic)
- In the flat case: 18 possible topologies ($\mathbb{R}^3$ is just one of them)

Cosmic Topology has **observable** effects

- The simplest case is the flat 3–torus
- The Universe repeats itself in every direction
- Defined by three vectors, pointing to your “clones” (6 physical dof)
- The repetition size must be large (larger than the CMB)
- So does it affect the observable Universe?
    - **Discretization of modes**
    - Correlation between modes with the same $|\vec{k}|$

Credit: COMPACT Collaboration



$$\langle \delta(\vec{k})\delta^*(\vec{k}')\rangle \propto P(|\vec{k}|)\ \delta^D(\vec{k}-\vec{k}')\ \text{for allowed}\ \vec{k}$$

- What are the main cosmological observables?
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Takeaway points

- Minkowski Functionals are useful tools to study fields **beyond Gaussianity** and **Isotropy**, with many applications in both the Early and Late Universe

- We have expanded the formalism to **CMB polarization** in two ways: with the polarization intensity P^2, and with the full information in the spin map

- We have created **Pynkowski** to ease the application of MFs to the community

- It is important to test the **isotropy** of the Universe for many reasons, such as to understand its **topology**

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Thank you!

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