LIGHT CURVES VARIATIONS AND ELEMENTS OF CW CASSIOPEIAE

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RIASSUNTO. — La variabile ad eclisse CW Cas è stata osservata fotoelettricamente dal 1957 al 1961. Considerando anche le misure ottenute nel 1956 si sono ricavate quattro curve di luce tutte diverse tra loro. La variazione non è limitata ad una sola parte della curva di luce, fenomeno spiegabile con l'esistenza di una zona perturbata nel sistema, ma è presente ovunque con andamento all'incirca opposto per i due minimi le cui profondità variano in senso contrario dello stesso importo. Le tredici epoche di minimo ottenute permettono di migliorare l'effemeride. La rappresentazione delle epoche più antiche, non fotoelettriche, richiede tuttavia un termine del secondo ordine. Gli elementi del sistema sono infine dedotti col metodo di Russell Merrill dalle curve di luce ottenute nel 1958-59 essendo apparsa non adatta la curva ottenuta nel 1956.

ABSTRACT. — Three light curves of the contact system CW Cas are derived from about 900 photoelectric observations obtained from 1957 to 1961. Looking upon also the measures executed with the same photometer in 1956 the system appears to change from one season to the other in all the parts of the light curves but with contrary trend during the two eclipses. The central depths of the minima undergo moreover exactly opposed changes. The ephemeris computed from the photoelectric measures do not represent satisfactory the oldest photographic observations which demand a parabolic term. The elements are computed from the 1956 and 1958-59 light curves but only the last give consistent results.

From some hundreds of photoelectric measures obtained during the 1956 ([1]) the writer derived the light and color curves of CW Cas, a W UMa eclipsing system. The variable was observed at the λ_{eff} (photomultiplier + filter) 0.43 and 0.58 μ. The light maxima appeared a little inequal chiefly in the bleu light. From a measure of the colors *B-V* and *U-B* it was deduced that the system is composed of two K0-K1 dwarfs. The C.I. resulted quite constant outside the eclipses while in the minima it was redder by about $0^m.05$. The observations reduced to the same phase showed a dispersion rather strong partly with a systematic trend so it can be supposed some little change in the system was occurred in the meantime. The trend of the period then resulted unsettled

(*) Ricevuta il 23 ottobre 1963.

for it was difficult to extimate the precision of the epochs of the minima non photoelectric since the corresponding measures are not published. Therefore it was judged advisable to observe during some years for explaining the period trend, and detecting eventual light curves mutations. Moreover it was hoped that a set of normal points obtained during a short time of « stability » for the system can permit the computation of the elements which for systems undergoing partial eclipses like this give usually considerable difficulty.

The Observations

The measures of this note were done, like the preceding ones, at the 40-inch reflector of the Merate Observatory with a Lallemand photomultiplier, and a BG 12 + GG 13 (1 mm) filter during the 1957 and from 1958 to 1961 with a BG 12 + GG 13 (2 mm) which is close to the *B* filter of the *U, B, V* system. As the color curve was determined in the 1956 it was preferred to measure only with the bleu filter for having more accurate light curves and so to define the eventual variations with greater accuracy. CW Cas was compared with the star *a* and this sometimes with the star *b* considered in the previous work ([1]). It is resulted:

1956	$\Delta m = m_a - m_b = + 0^m.38$	; average deviation of a Δm :	$\pm\ 0^m.007$
1957	$= + 0^m.376$	» » » »	$\pm$ .008
1958-61	$= + 0^m.392$	» » » »	$\pm$ .006

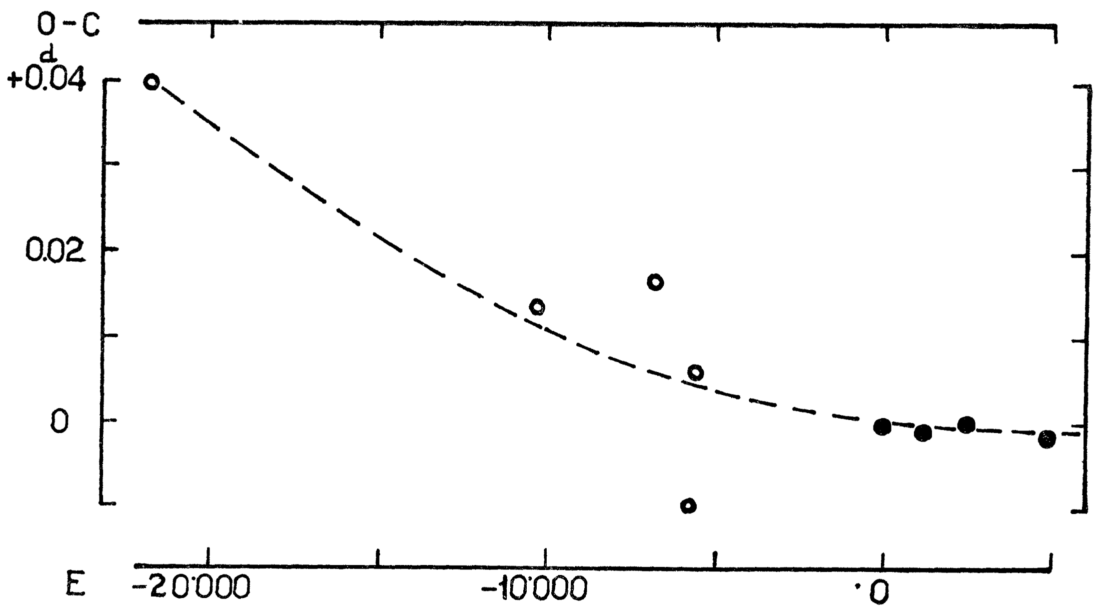

Fig. 1. — Residuals from the linear ephemeris (1) of the epochs of minimum. The dots refer to the photoelectric observations, the circles to the photographic.

Remembering the little change in the filters and that the two stars differ $0^m.5$ in the color index it can be concluded the comparisons were constant. The individual differential magnitudes $\Delta m = m_{var} - m_a$, in the total number of 924, are given in the Table I, corrected for the differential extinction.

EPOCH AND PERIOD

Seven epochs of Min I and six of Min II have been determined altogether, given in the Table II after the epochs obtained in 1956 and those reported in the references [1]. Some values deduced from our light curves with no hight precision are given with three decimal figures. Assigning half weight to these, from the photoelectric epochs alone we have computed two ephemeris separately for the principal and secondary minima. As the periods coincide within their mean errors, combining all the epochs it is obtained:

$$(1) \qquad \text{Min I} = \text{helioc. J.D. } 2435745.6163 + 0.31886497 \text{ E}$$
$$\pm \quad 3 \quad \pm \quad 11 \text{ e.m.}$$

The residuals of the photoelectric epochs, one minute on an average, shown a slight systematic trend in the interval J.D. 36544—37343. Considering also the $O - C$ of the photographic epochs it appears, as already remarked, the period is probably not constant. Since the distri-

TABLE II. — *Epochs of minimum light.*

Observer		E	Helioc. J.D. 24.....	O—C	Observer		E	Helioc. J.D. 24.....	O—C
S	*ph*	— 21781.5	28800.30	^{d} + 0.04	B	*pe*	+ 1223	36135.587	— 0.001
K	»	10282	32467.061	+ .014	»	»	1226	6136.5450	+ .0002
K	»	6829.5	3567.945	+ .017	»	»	1228.5	6137.3418	— .0001
K	»	5694	3929.989	— .010	»	»	2505	6544.3740	+ .0010
K	»	5643.5	3946.108	+ .006	»	»	2530	6552.3450	+ .0003
B	*pe*	— 0.5	5745.4567	— .0002	»	»	2530.5	6552.5047	+ .0006
»	»	0	5745.6156	— .0007	»	»	2596	6573.3901	+ .0003
»	»	+ 5.5	5747.3711	+ .0010	»	»	2599	6574.3469	+ .0005
»	»	6	5747.5280	— .0015	»	»	2611.5	6578.3339	+ .0017
»	»	31	5755.4995	— .0016	»	»	4747.5	7259.427	— .001
»	»	37	5757.4146	+ .0003	»	»	4935.5	7319.3742	— .0002
»	»	37.5	5757.5736	— .0001	»	»	4976	7332.2879	— .0005
»	»	209	5812.261	+ .002	»	»	+ 5010.5	37343.2880	— 0.0012
»	»	+ 216	35814.4920	+ 0.0009					

S = Sverev K = Kaho B = Broglia
ph = photographic *pe* = photoelectric

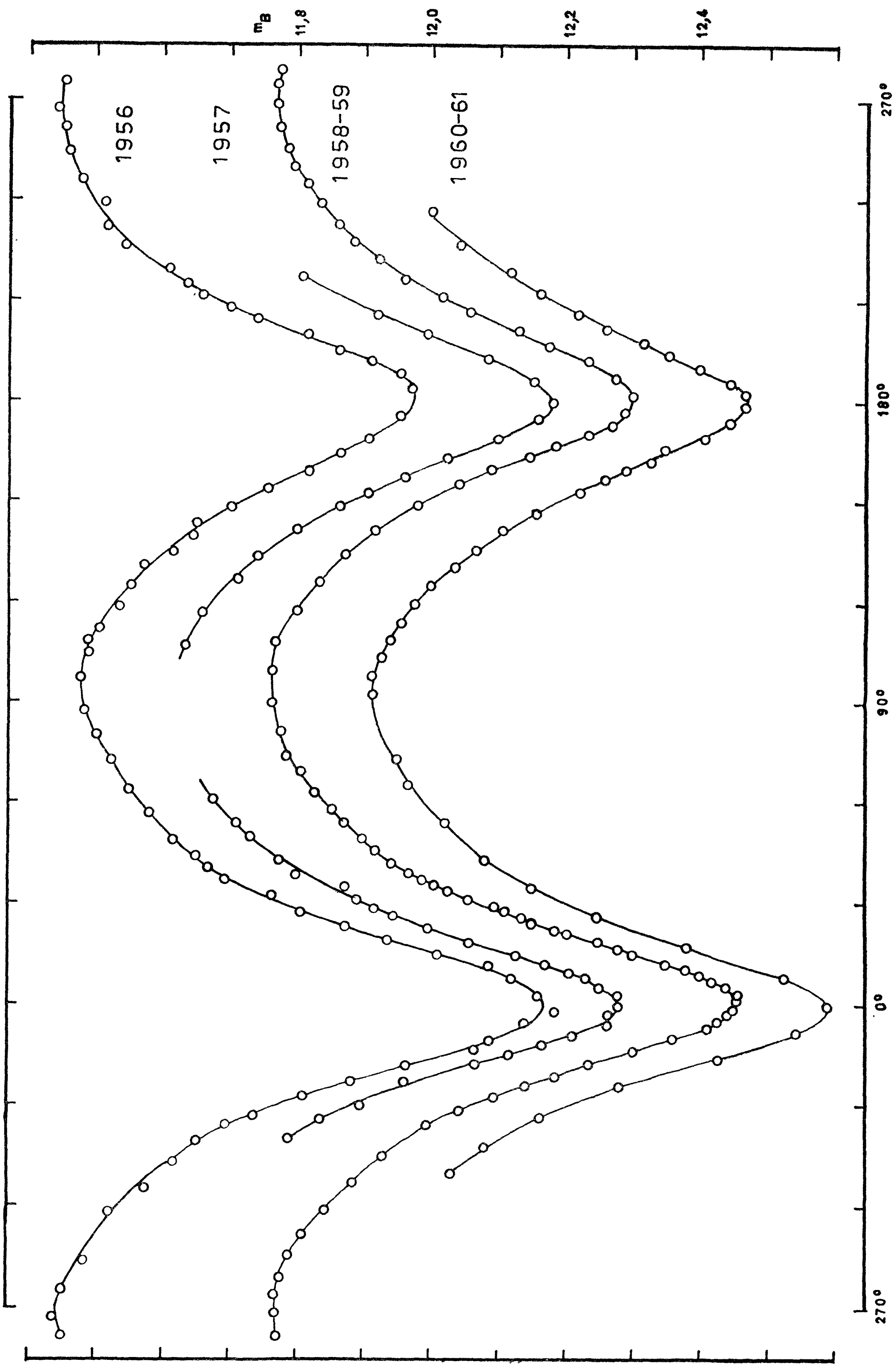

Fig. 2. — Light curves (λ = 0.43 μ) of CW Cas during the years 1956, 1957, 1958-59 and 1960-61. Each curve is shifted vertically of $0^m.15$ with respect to the contiguous. The vertical scale refers to the 1958-59 light curve from which the elements are computed.

bution of the epochs is unfavourable it is premature to judge the trend of the variation which at first sight look parabolic (Fig. 1) and only future observations will define. The phases of each Δm have been computed (Table I) by:

(2) helioc. phase = (helioc J.D. — 2435000) 3.1361237

For the primary minimum we have the mean phase: 0.3450±0.0009 e.m., for the secondary: 0.8452 ± 0.0009 e.m. They are far off $0^{P}.5002$ ± 0.0013 e.m.; therefore the tangential component of the orbital eccentricity is null.

DISCUSSION AND ELEMENTS

The measures of each observing season, which when reported in phase do not show any systematic deviation between different nights, excepting a little the first, have been collected in the following groups:

Interval J.D.	Measures	Normal points
2435745 — 786	275	65
6135 — 137	199	45
6544 — 609	544	81
7259 — 343	181	45

The phases of the Δm bleu of 1956 ([1]) have been recomputed throught the (2). The normal points are given in the Tables III, IV, V, VI and the corresponding light curves are reproduced in the Fig. 2. They appear clearly variable everywhere but a little more in the secondary minimum. The variation is roughly opposite during the two eclipses and the depths of the minima undergo exactly contrary variations (Fig. 3). It is known that the fluctuations of the light curve are quite usual for the W UMa variables but only for a few systems it is a homogeneous set of observations obtained in different seasons with the same photometer. Referring for instance to three of such variables, U Peg, AH Vir ([3]) and RZ Com ([4]) it can be seen that the variations affect only a part of the light curve so it can be supposed the existence of a disturbed local region on one component. For CW Cas instead the perturbing region seems more extended and the opposite variation of the depths of the minima, which as suggest the Fig. 3 can have a periodic trend, hint a sort of stream or ring with not uniform opacity which moves slowly around the system. The photometric data cannot specify more and a better insight of the problem demand a more extended observational basis.

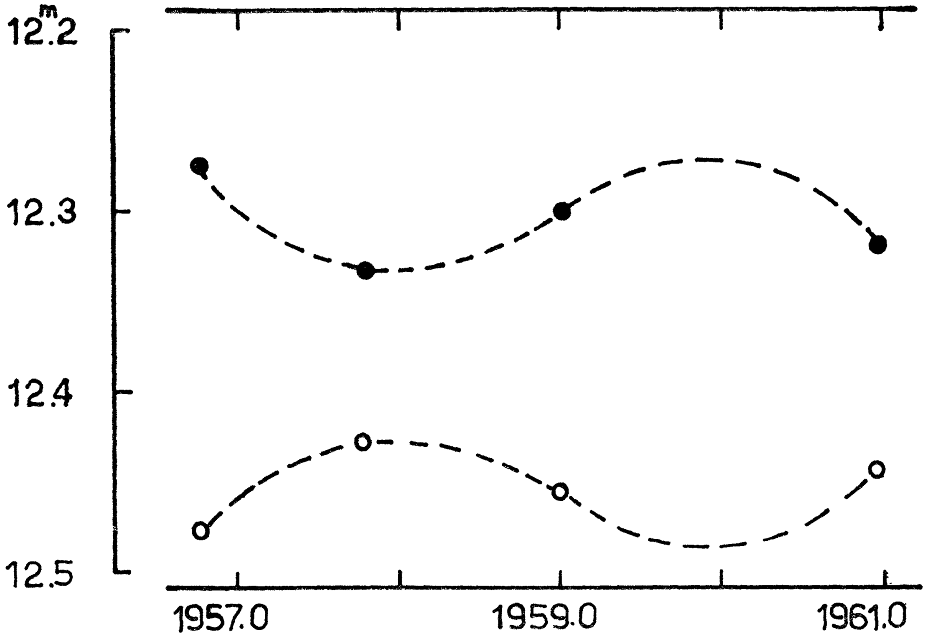

Fig. 3. — The opposite fluctuations of the depths of the primary minimum (below) and secondary (above). The broken lines do not indicate a periodicity which cannot be proved from these few minima but underline only the opposite trend.

TABLE III. — *Normal points for the interval J.D.* 2435745—786

n	Phase	Δ m	n	Phase	Δ m	n	Phase	Δ m
	°			°			°	
5	2.6	+ 0.839	4	124.4	+ 0.233	5	232.1	+ 0.195
5	7.8	.798	4	130.9	.251	5	239.0	.191
4	11.3	.767	5	135.1	.295	4	245.8	.156
4	15.0	.688	5	139.7	.325	4	254.3	.136
4	19.0	.617	5	143.5	.330	4	260.8	.132
4	22.7	.552	5	147.9	.380	4	266.8	.120
4	27.1	.483	4	154.0	.434	4	275.1	.130
4	31.7	.442	4	159.3	.497	4	283.7	.164
4	36.5	.374	4	164.3	.544	4	297.6	.200
4	40.1	.348	4	168.8	.587	4	305.3	.255
4	43.6	.331	4	175.4	.634	4	312.7	.299
4	48.6	.296	4	183.4	.652	4	319.3	.332
4	56.1	.258	4	188.1	.633	4	324.2	.376
4	63.6	.230	4	191.8	.593	4	327.0	.417
4	71.8	.207	4	195.2	.542	4	332.8	.489
4	80.0	.182	4	200.2	.495	4	336.8	.560
4	86 9	.162	4	204 5	.421	4	342.0	.644
5	97.3	.159	4	207.6	.378	4	346.5	.745
5	104.5	.172	4	211.5	.337	4	349.3	.768
5	108.4	.168	4	214.8	.314	4	354.4	.818
5	112.2	.185	5	219.4	.288	5	358.1	.865
4	118.6	.214	5	226.3	.222			

TABLE IV. — *Normal points for the interval J.D.* 2436135—137

n	Phase	Δ m	n	Phase	Δ m	n	Phase	Δ m
	°			°			°	
3	2.7	+ 0.808	4	53.8	+ 0.242	6	192.8	+ 0.615
4	5.0	.778	4	60.4	.206	6	200.0	.525
4	7.7	.759	4	106.8	.164	5	206.0	.449
4	9.4	.736	6	116.4	.191	6	217.3	.335
4	11.8	.699	6	126.5	.241	4	320.1	.318
3	14.3	.655	5	133.5	.271	3	325.7	.366
4	18.2	.586	5	141.6	.330	4	330.0	.424
4	22.6	.526	4	148.4	.394	4	337.2	.491
4	26.0	.473	4	152.5	.437	4	342.0	.596
5	28.5	.445	5	157.3	.492	3	344.9	.646
4	31.0	.420	5	162.9	.553	3	348.1	.695
6	35.2	.402	5	168.8	.628	4	350.5	.739
5	38.3	.330	5	174.4	.688	3	353.7	.792
6	42.5	.305	5	179.6	.712	4	357.0	.793
4	49.4	.261	6	186.0	.682	3	359.5	.808

TABLE V. — *Normal points for the interval J.D.* 2436544—609.

n	Phase	Δ m	n	Phase	Δ m	n	Phase	Δ m
	°			°			°	
7	0.7	+ 0.834	7	69.0	+ 0.186	7	227.7	+ 0.264
7	2.9	.837	7	73.8	.163	7	232.8	.241
8	5.0	.819	6	80.9	.157	7	238.7	.213
7	7.1	.798	6	90.1	.144	7	244.5	.194
7	8.7	.779	7	99.5	.143	7	250.1	.173
7	10.5	.759	6	107.9	.148	7	255.3	.163
7	12.0	.728	6	117.6	.181	7	261.7	.152
8	14.6	.680	6	125.7	.212	7	268.7	.149
8	16.6	.658	5	133.8	.252	7	274.2	.147
7	18.5	.629	5	141.1	.297	7	278.9	.155
7	20.8	.583	5	148.9	.360	7	285.6	.169
7	22.1	.564	5	155.3	.422	7	291.5	.187
7	24.0	.531	5	159.4	.468	6	298.9	.222
7	25.7	.515	5	163.1	.525	6	306.7	.265
7	27.6	.491	5	166.5	.565	6	315.3	.308
7	29.2	.475	5	169.9	.613	6	324.1	.375
7	31.1	.435	5	173.2	.649	7	328.3	.423
7	33.5	.407	6	176.6	.668	7	332.5	.475
8	35.2	.387	7	181.7	.680	7	335.7	.522
7	37.0	.370	7	187.2	.653	7	338.6	.565
7	39.0	.348	7	192.0	.614	8	342.0	.616
7	42.2	.321	7	196.6	.555	7	346.1	.680
7	45.5	.296	7	200.9	.510	7	350.0	.739
7	49.3	.279	7	206.8	.439	7	352.8	.792
7	53.9	.251	7	211.1	.397	7	354.8	.805
7	57.8	.233	7	216.7	.341	7	357.0	.822
7	63.0	.207	7	222.7	.301	7	358.7	.829

TABLE VI. — *Normal points for the interval J.D.* 2437259—343.

n	Phase	Δ m	n	Phase	Δ m	n	Phase	Δ m
3	8.1°	+ 0.755	4	130.5°	+ 0.265	5	194.1°	+ 0.582
3	17.3	.610	5	135.6	.298	5	197.8	.544
3	25.9	.477	5	141.6	.336	5	201.9	.490
3	34.3	.380	5	147.3	.386	4	206.3	.447
3	42.7	.312	5	152.8	.449	4	212.0	.391
2	53.9	.251	4	156.7	.487	3	218.8	.347
3	64.9	.197	4	159.3	.518	3	226.4	.272
3	72.5	.179	5	162.0	.555	4	236.7	.228
6	92.5	.142	4	165.5	.577	3	309.8	.260
5	97.7	.141	4	169.2	.636	3	317.4	.310
5	103.5	.157	5	173.7	.672	3	326.2	.392
5	108.8	.168	5	178.4	.696	3	335.6	.508
5	113.4	.184	4	182.3	.697	3	344.2	.656
5	119.4	.206	4	185.4	.674	3	351.6	.772
5	125.1	.232	5	190.2	.628	3	359.8	.820

The computation of the elements has been performed according to the Russell Merrill method ([2]), to which formulae we refer after, separately for the 1956 and 1958-59 light curves, the only suitable among the four sets of observations. The harmonic analysis by least squares of the normals between the minima, and the rectification also, was done with the IBM 1620 of the Milano Observatory in the form: $l_{obs} = A_0 + \Sigma A_n \cos n \vartheta + \Sigma B_m \operatorname{sen} m \vartheta$, where $n \leqslant 2$, $m \leqslant 6$. After several trials made for identifying the correct value of the external contact angle ϑ' and to exclude the terms not consistent, we have obtained the values of the Table VII. The coefficients of the term in $\cos \vartheta$ of the two groups have the correct sign and are a little greater than should be for stars of nearly equal surface temperature. Those of $\cos 2\vartheta$ are equal, after normalisation to the unity of the light out of the eclipse. Since the reflection influence little this term it means that the oblateness of the stars is not changed. The terms in $\operatorname{sen} n\vartheta$ vary considerably as it appears also from the respective light curves which undergo different distorsions of greater amount for the older one. After subtraction to the normals of the terms in $\operatorname{sen} n\vartheta$ the asymmetries disappear completely outside the eclipses and in the primary minimum of both light curves; in the secondary they are considerably reduced but do not cancel completely everywhere. The rectification of the reflection and ellipticity of the components were performed in the usual manner with $C_0 = 0.0410$ and $C_2 = 0.0137$. The phases not rectified of the external contacts are for the Min I and the Min II respectively 40°, 44° (1956); 43°, 44° (1958-59). The different length of the eclipses was judged not significant and in account of the residuals of the epochs of minimum a circular orbit was assumed. The average deviation from the unity of a rectified normal outside the eclipses resulted ± 0.0053 and ± 0.0017 respectively for the 1956 and 1958-59 groups and the corresponding rectified lights at $\vartheta = 0°$: 0.744, 0.746; at $\vartheta = 180°$: 0.793, 0.790. The rectification of the phases has been computed with $Nz = 0.463$, which resulted the same for the two groups and with $x = 0.8$ in account of the λ_{eff} and the spectral type of the components.

TABLE VII. — *Coefficients of the harmonic analysis of the outside eclipse variations.*

	1956 (27 equations)		1958-59 (29 equations)	
A_0	+ 0.7684 ±	0.0023 e.m.	+ 0.8774 ±	0.0010 e.m.
A_1	— .0299	26	— .0216	10
A_2	— .1065	35	— .1223	14
B_1	— .0136	14	+ .0046	5
B_2	+ 0.0052 ±	0.0016	— .0039	6
B_5	—		— .0024	6
B_6	—		— 0.0014 ±	0.0006

TABLE VIII. — *Elements derived from the 1958-59 light curve.*

x (assumed)	0.8	a_g	0.459 ± 0.013
Min I	transit	a_s	0.292 .015
k	0.635 ± 0.052	L_g	0.749 .027
α_0^{tr}	0.74	L_s	0.251 .027
α_0^{occ}	0.84	J_s/J_g	0.832 ± 0.014
p_0	— 0.58 ± 0.13	Max	$11^m.76$
θ_e	$46°.4$	Min I	$12^m.45$
i	$73° \pm 1°$	Min II	$12^m.30$

The identification of the type of the eclipses by means of the χ ($n = 0.8$) values resulted unsuccessful for both the light curves. Owing to the perturbations remaining after the rectification in the secondary minimum the elements were derived only from the primary. Computing some nomographic solutions with the χ functions, moving along the depht line, the best representation of the normals was obtained supposing the Min I is a transit. The older curve moreover on account of the dispersion of the normals gives no more precise indication excepting the k value lies between 0.9 (occultation hypothesis) and 0.5 (transit). The 1958-59 measures permit with the same procedure to restrict the acceptable range of k from 0.90 (transit) to 0.55 (transit). For a conclusive solution only the last group appeared suitable. Chosing three values of k distributed along the precedent interval the corresponding α_0^{tr} was determined from the depth line and $sen^2 \vartheta$ $(n = 0.5)$ from the light curve. Then the light curve and the weighed sum of the squares of the residuals $\Sigma\, w\, (O - C)^2$ of the rectified normals were computed by means of the Ψ^{tr} Tables for each of the three cases, with $\sqrt{w} = \frac{1}{l}\,\frac{\Delta\, l}{\Delta\, sen^2\, \Theta}$. Assuming the more probable value of k corresponds to the minimum of the sum of the squares of the residuals, such value is the abscissa of the top of the parabola determined by the three pairs $\Sigma\, w\, (O - C)^2$, k and its mean error ([4]) is equal to $(k - k')$ where k' is pertinent to the $\Sigma\, w\, (O - C)^2$ equal to $n/(n - 1)$ times its minimum value (n = number of normals). Because the solution depends on the extimed $sen^2\, \vartheta$ $(n = 0.5)$ the precedent computations were repeated for three different values of this quantity for obtaining, with the above mentioned method, the best representation. The effect of assuming a different darkening coefficient, 0.6 instead of 0.8, was found negligible. The solution depends moreover on the precision of the dephts of the eclipses but this effect, according to the accuracy of the minima, give a minor contribute to the mean error of the geometrical elements. In Table VIII the final results are given. The mean residual for a normal used in the solution is 0.0029 like the intrinsec mean error of a normal near the quarter points. The external contact θ_e correspond to the not rectified value $\vartheta' = 44^\circ.2$, the same extimed from the light curve. The inclination corrected for the polar flattening with $z = 0.145$ is $74^\circ.5$. The sum $a_s + a_g$ as well as the ratio k is consistent with the Roche model for a contact binary.

REFERENCES

(1) Merate Contr. N. 113 (1957).
(2) Princeton Contr. N. 26 (1952).
(3) Flower and Cook Obs. Reprint N. 120 (1960), N. 118 (1959).
(4) Merate Contr. N. 165 (1960).
(5) B.A.N. VIII, 141 (1937).

TABLE I. — *Observations of CW Cas.*

Helioc. J.D. 2436000.+	Phase	Δ m	Helioc. J.D. 2436000.+	Phase	Δ m
135.5739	0.3002	+ 0.628	136.5249	.2827	0.529
.5778	.3125	.699	.5270	.2893	.553
.5783	.3140	.722	.5275	.2908	.572
.5793	.3172	.752	.5279	.2921	.585
.5828	.3281	.819	.5298	.2980	.608
.5864	.3394	.792	.5302	.2993	.618
.5871	.3416	.806	.5308	.3012	.647
.5904	.3520	.819	.5329	.3078	.662
.5910	.3538	.803	.5334	.3093	.664
.5932	.3607	.774	.5359	.3172	.720
.5945	.3648	.767	.5365	.3191	.735
.5950	.3664	.756	.5370	.3206	.752
.5955	.3680	.760	.5388	.3263	.772
.5974	.3739	.717	.5395	.3285	.786
.5980	.3758	.706	.5406	.3319	.796
.5996	.3808	.684	.5423	.3372	.790
.6002	.3827	.676	.5428	.3388	.795
.6044	.3959	.579	.5444	.3438	.815
.6049	.3974	.569	.5448	.3451	.805
.6060	.4009	.562	.5470	.3520	.803
.6105	.4150	.490	.5480	.3551	.787
.6115	.4181	.466	.5493	.3592	.784
.6132	.4235	.448	.5499	.3611	.767
.6139	.4257	.451	.5517	.3667	.754
.6166	.4341	.406	.5523	.3686	.751
.6178	.4379	.409	.5527	.3699	.745
.6202	.4454	.355	.5532	.3714	.730
.6208	.4473	.344	.5547	.3761	.707
.6245	.4589	.303	.5553	.3780	.698
.6250	.4605	.298	.5572	.3840	.648
.6255	.4620	.296	.5583	.3874	.642
.6259	.4633	.294	.5603	.3937	.602
.6323	.4834	.248	.5608	.3953	.594
.6339	.4884	.240	.5647	.4075	.522
.6344	.4900	.241	.5652	.4091	.521
.6350	.4918	.245	.5665	.4131	.498
.6400	.5075	.216	.5670	.4147	.485
.6407	.5097	.208	.5690	.4210	.453
.6428	.5163	.201	.5695	.4225	.446
135.6434	.5182	+ 0.201	.5700	.4241	.442
			.5705	.4257	.438
136.5070	0.2265	+ 0.315	.5715	.4288	.436
.5096	.2347	.323	.5720	.4304	.419
.5103	.2369	.318	.5724	.4316	.418
.5108	.2385	.315	.5752	.4404	.471
.5138	.2479	.356	.5758	.4423	.469
.5144	.2497	.365	.5763	.4439	.363
.5150	.2516	.378	.5776	.4479	.354
.5164	.2560	.404	.5781	.4495	.342
.5183	.2620	.425	.5788	.4517	.333
.5188	.2635	.430	.5809	.4583	.317
.5193	.2651	.437	.5815	.4602	.322
.5219	.2733	.467	.5830	.4649	.312
.5245	.2814	.418	.5837	.4671	.312

Table I (continued)

Helioc. J.D. 2436000.+	Phase	Δ m	Helioc. J.D. 2436000.+	Phase	Δ m
136.5870	.4774	0.278	137.3344	.8214	0.675
.5876	.4793	.278	.3367	.8286	.675
.5933	.4972	.241	.3372	.8301	.683
136.5938	.4988	+ 0.239	.3376	.8314	.693
			.3391	.8361	.716
137.2755	0.6366	+ 0.153	.3396	.8377	.705
.2762	.6388	.162	.3400	.8389	.730
.2769	.6410	.169	.3424	.8465	.710
.2800	.6508	.172	.3428	.8477	.705
.2817	.6561	.179	.3433	.8493	.710
.2831	.6605	.179	.3449	.8543	.703
.2860	.6696	.200	.3454	.8559	.704
.2867	.6718	.198	.3474	.8621	.684
.2872	.6733	.194	.3479	.8637	.663
.2892	.6796	.196	.3483	.8650	.669
.2926	.6903	.229	.3496	.8690	.668
.2930	.6915	.233	.3502	.8709	.657
.2936	.6934	.240	.3523	.8775	.614
.2956	.6997	.239	.3528	.8791	.615
.2961	.7013	.248	.3544	.8841	.609
.2965	.7025	.258	.3549	.8857	.597
.2970	.7041	.266	.3554	.8872	.600
.3009	.7163	.276	.3581	.8957	.555
.3014	.7179	.272	.3586	.8973	.541
.3020	.7198	.174	.3590	.8985	.531
.3024	.7210	.267	.3603	.9026	.518
.3062	.7329	.306	.3608	.9042	.498
.3068	.7348	.304	.3613	.9057	.504
.3084	.7398	.343	.3633	.9120	.464
.3089	.7414	.348	.3637	.9133	.456
.3094	.7430	.347	.3649	.9170	.451
.3129	.7539	.382	.3661	.9208	.434
.3134	.7555	.390	.3666	.9223	.438
.3139	.7571	.394	.3705	.9346	.361
.3155	.7621	.410	.3718	.9387	.358
.3160	.7637	.416	.3732	.9430	.340
.3164	.7649	.423	.3776	.9568	.323
.3187	.7721	.452	.3781	.9584	.316
.3192	.7737	.455	137.3785	.9597	+ 0.313
.3197	.7753	.465			
.3211	.7797	.482	544.3432	0.2513	+ 0.382
.3216	.7812	.492	.3450	.2570	.420
.3222	.7831	.501	.3455	.2585	.429
.3244	.7900	.522	.3475	.2648	.458
.3249	.7916	.540	.3482	.2670	.475
.3255	.7935	.548	.3503	.2736	.502
.3273	.7991	.556	.3510	.2758	.510
.3277	.8004	.554	.3517	.2780	.518
.3283	.8022	.568	.3532	.2827	.556
.3308	.8101	.594	.3536	.2839	.572
.3312	.8113	.608	.3542	.2858	.578
.3317	.8129	.619	.3563	.2924	.601
.3327	.8160	.649	.3569	.2943	.618
.3339	.8198	.669	.3617	.3093	.669

Table I (continued)

Helioc. J.D. 2436000.+	Phase	Δ m	Helioc. J.D. 2436000.+	Phase	Δ m
544.3659	.3225	0.803	545.3507	.4110	0.530
.3673	.3269	.816	.3513	.4128	.509
.3693	.3332	.800	.3519	.4147	.500
.3712	.3391	.839	.3539	.4210	.499
.3764	.3554	.851	.3544	.4226	.485
.3780	.3604	.845	.3551	.4248	.475
.3786	.3623	.837	.3569	.4304	.445
.3802	.3673	.796	.3576	.4326	.433
.3807	.3689	.795	.3606	.4420	.392
.3826	.3749	.777	.3611	.4436	.387
.3832	.3768	.761	.3617	.4454	.384
.3851	.3827	.707	.3627	.4486	.360
.3868	.3880	.690	545.3637	.4517	+ 0.347
.3873	.3896	.682			
.3893	.3959	.659	552.3361	0.3180	+ 0.749
.3898	.3975	.635	.3376	.3227	.769
.3915	.4028	.587	.3391	.3274	.795
.3921	.4047	.590	.3409	.3331	.820
.3937	.4097	.551	.3414	.3347	.816
.3942	.4113	.538	.3428	.3390	.843
.3961	.4172	.527	.3434	.3409	.838
.3966	.4188	.511	.3450	.3459	.841
.3984	.4244	.509	.3456	.3478	.850
.4000	.4294	.472	.3471	.3525	.842
.4005	.4310	.456	.3475	.3538	.845
.4011	.4329	.406	.3492	.3591	.816
.4030	.4389	.412	.3496	.3604	.805
.4037	.4410	.403	.3511	.3651	.791
.4052	.4458	.392	.3516	.3666	.780
.4070	.4514	.362	.3529	.3707	.772
.4079	.4542	.335	.3532	.3717	.751
.4101	.4611	.331	.3539	.3739	.747
544.4117	.4661	+ 0.333	.3553	.3782	.718
			.3560	.3804	.707
545.3242	0.3278	+ 0.817	.3577	.3858	.677
.3253	.3313	.824	.3580	.3867	.661
.3272	.3373	.830	.3600	.3930	.645
.3278	.3391	.826	.3605	.3946	.637
.3297	.3451	.839	.3633	.4033	.579
.3303	.3470	.829	.3636	.4043	.568
.3320	.3523	.834	.3659	.4115	.544
.3324	.3536	.821	.3675	.4165	.516
.3342	.3592	.813	.3680	.4181	.514
.3351	.3620	.825	.3696	.4231	.477
.3369	.3677	.792	.3700	.4243	.466
.3387	.3733	.789	.3717	.4297	.435
.3392	.3749	.772	.3722	.4312	.443
.3412	.3812	.726	.3735	.4353	.423
.3432	.3874	.665	.3741	.4372	.419
.3453	.3940	.632	.3761	.4435	.391
.3458	.3956	.623	.3770	.4463	.381
.3477	.4016	.591	.3775	.4479	.373
.3483	.4034	.591	.3793	.4535	.356
.3488	.4050	.566	.3798	.4551	.346

Table I (continued)

Helioc. J.D. 2436000.+	Phase	Δ m	Helioc. J.D. 2436000.+	Phase	Δ m
552.3823	.4629	0.324	552.4522	.6821	0.191
.3842	.4689	.312	.4530	.6846	.193
.3850	.4714	.303	.4548	.6903	.200
.3867	.4767	.298	.4553	.6919	.211
.3872	.4783	.293	.4559	.6937	.207
.3889	.4836	.281	.4582	.7010	.225
.3894	.4852	.274	.4589	.7031	.234
.3914	.4915	.261	.4595	.7050	.234
.3920	.4933	.255	.4620	.7129	.244
.3926	.4952	.250	.4627	.7151	.248
.3944	.5009	.247	.4657	.7245	.262
.3949	.5024	.241	.4662	.7260	.273
.3955	.5043	.240	.4667	.7276	.271
.3978	.5115	.221	.4690	.7348	.288
.3985	.5137	.217	.4696	.7367	.296
.4014	.5228	.209	.4712	.7417	.314
.4021	.5250	.209	.4720	.7442	.317
.4053	.5351	.197	.4741	.7508	.332
.4059	.5369	.194	.4750	.7536	.339
.4064	.5385	.191	.4772	.7605	.373
.4082	.5441	.179	.4779	.7627	.377
.4089	.3463	.179	.4786	.7649	.379
.4109	.5526	.168	.4807	.7715	.408
.4115	.5545	.154	.4815	.7740	.419
.4134	.5605	.170	.4822	.7762	.417
.4139	.5620	.157	.4839	.7816	.446
.4144	.5636	.159	.4846	.7837	.451
.4175	.5733	.158	.4854	.7863	.464
.4184	.5761	.149	.4872	.7919	.503
.4207	.3833	.149	.4879	.7941	.511
.4214	.5855	.151	.4885	.7960	.515
.4220	.5874	.131	.4907	.8029	.559
.4240	.5937	.144	.4913	.8048	.570
.4246	.5956	.155	.4920	.8070	.578
.4272	.6037	.146	.4938	.8126	.601
.4278	.6056	.141	.4943	.8142	.606
.4298	.6119	.144	.4949	.8161	.610
.4303	.6135	.141	.4968	.8220	.645
.4309	.6153	.143	.4975	.8242	.658
.4331	.6222	.150	.4981	.8261	.661
.4337	.6241	.138	.4987	.8280	.659
.4356	.6301	.140	.5005	.8336	.675
.4362	.6320	.146	.5011	.8355	.678
.4368	.6338	.139	.5016	.8371	.686
.4387	.6398	.148	.5039	.8443	.687
.4393	.6417	.143	.5046	.8465	.678
.4408	.6464	.152	.5067	.8531	.682
.4426	.6520	.154	.5072	.8546	.678
.4436	.6552	.156	.5087	.8593	.664
.4454	.6608	.166	.5093	.8612	.657
.4460	.6627	.171	.5111	.8669	.656
.4494	.6734	.184	.5117	.8687	.654
.4500	.6752	.188	.5135	.8744	.632
.4506	.6771	.187	.5141	.8763	.628

Table I (continued)

Helioc. J.D. 2436000.+	Phase	Δ m
552.5146	.8778	0.621
.5162	.8828	.593
.5168	.8847	.587
.5174	.8866	.579
.5192	.8923	.550
.5198	.8941	.545
.5204	.8960	.532
.5223	.9020	.516
.5229	.9039	.499
.5236	.9061	.490
.5242	.9079	.492
.5250	.9104	.487
.5275	.9183	.443
.5281	.9202	.438
.5288	.9224	.433
.5305	.9277	.428
.5313	.9302	.413
.5320	.9324	.399
.5347	.9409	.366
.5356	.9437	.349
.5363	.9459	.343
.5389	.9540	.324
552.5421	.9641	+ 0.299
573.3612	0.2553	+ 0.443
.3647	.2663	.457
.3673	.2745	.512
.3677	.2757	.536
.3693	.2808	.536
.3700	.2829	.543
.3722	.2898	.586
.3726	.2911	.591
.3746	.2974	.620
.3752	.2993	.630
.3779	.3077	.677
.3785	.3096	.686
.3807	.3165	.741
.3812	.3181	.748
.3824	.3218	.765
.3844	.3281	.792
.3849	.3297	.799
.3865	.3347	.816
.3871	.3366	.821
.3891	.3428	.834
.3897	.3447	.823
.3903	.3466	.828
.3909	.3485	.821
.3935	.3566	.821
.3943	.3592	.810
.3959	.3642	.787
.3966	.3664	.776
.3990	.3739	.750
.3996	.3758	.733
.4003	.3780	.727
573.4026	.3852	0.689
.4033	.3874	.679
.4041	.3899	.668
.4048	.3921	.645
.4072	.3996	.600
.4080	.4021	.576
.4087	.4043	.562
.4108	.4109	.541
.4115	.4131	.534
.4123	.4156	.517
.4138	.4203	.499
.4146	.4228	.486
.4155	.4256	.483
.4193	.4376	.404
.4200	.4398	.398
.4206	.4416	.385
.4228	.4485	.366
.4235	.4507	.356
.4243	.4532	.357
.4280	.4648	.296
.4285	.4664	.294
573.4307	.4733	+ 0.289
574.2204	0.9499	+ 0.328
.2227	.9571	.315
.2247	.9634	.306
.2274	.9719	.276
.2279	.9734	.280
.2297	.9791	.260
.2304	.9813	.252
.2311	.9835	.251
.2343	.9935	.245
.2368	.0013	.223
.2373	.0029	.223
.2393	.0092	.220
.2401	.0117	.222
.2421	.0180	.209
.2429	.0205	.213
.2437	.0230	.200
.2460	.0302	.177
.2467	.0324	.180
.2481	.0368	.173
.2488	.0390	.172
.2511	.0462	.160
.2516	.0478	.166
.2531	.0525	.167
.2541	.0556	.164
.2565	.0631	.168
.2573	.0656	.165
.2634	.0848	.151
.2654	.0910	.153
.2662	.0935	.150
.2668	.0954	.149
.2692	.1029	.158

Table I (continued)

Helioc. J.D. 2436000.+	Phase	Δ m	Helioc. J.D. 2436000.+	Phase	Δ m
574.2700	.1055	0.155	574.3352	.3099	0.702
.2724	.1130	.158	.3374	.3168	.731
.2731	.1152	.159	.3381	.3190	.743
.2736	.1167	.164	.3395	.3234	.774
.2793	.1346	.171	.3402	.3256	.778
.2811	.1403	.172	.3407	.3272	.789
.2818	.1425	.180	.3424	.3325	.810
.2835	.1478	.186	.3431	.3347	.815
.2842	.1500	.180	.3444	.3388	.819
.2847	.1516	.181	.3450	.3407	.819
.2854	.1538	.180	.3456	.3425	.826
.2878	.1613	.193	.3474	.3482	.831
.2884	.1632	.194	.3481	.3504	.834
.2890	.1650	.203	.3495	.3548	.833
.2910	.1713	.223	.3503	.3573	.827
.2915	.1729	.223	.3512	.3601	.821
.2930	.1776	.218	.3532	.3664	.792
.2936	.1795	.227	.3537	.3680	.780
.2955	.1854	.248	.3543	.3698	.770
.2961	.1873	.255	.3554	.3733	.749
.2967	.1892	.261	.3561	.3755	.732
.2987	.1955	.261	.3568	.3777	.726
.2994	.1977	.264	.3587	.3836	.689
.3018	.2052	.274	.3594	.3858	.679
.3024	.2071	.279	.3602	.3883	.662
.3031	.2093	.285	.3617	.3930	.642
.3046	.2140	.288	.3624	.3952	.629
.3070	.2215	.311	.3628	.3965	.624
.3079	.2243	.314	.3647	.4024	.590
.3086	.2265	.323	.3653	.4043	.578
.3098	.2303	.329	.3659	.4062	.568
.3125	.2387	.354	.3672	.4103	.539
.3131	.2406	.353	.3678	.4122	.527
.3145	.2450	.366	.3684	.4141	.523
.3147	.2456	.382	.3702	.4197	.502
.3153	.2475	.385	.3710	.4222	.489
.3173	.2538	.413	.3716	.4241	.470
.3179	.2557	.419	.3733	.4294	.451
.3186	.2579	.433	.3741	.4319	.432
.3200	.2623	.440	.3760	.4379	.408
.3207	.2645	.447	.3765	.4395	.390
.3227	.2707	.488	.3770	.4410	.386
.3235	.2732	.504	.3784	.4454	.373
.3250	.2779	.520	.3790	.4473	.365
.3256	.2798	.528	.3814	.4548	.336
.3274	.2855	.555	.3820	.4567	.330
.3282	.2880	.568	.3826	.4586	.330
.3296	.2924	.603	.3850	.4661	.306
.3301	.2939	.611	.3856	.4680	.300
.3318	.2993	.654	.3881	.4758	.281
.3324	.3012	.670	.3886	.4774	.284
.3330	.3030	.677	.3893	.4796	.281
.3341	.3065	.687	.3907	.4840	.274
.3347	.3084	.696	.3913	.4859	.271

Table I (continued)

Helioc. J.D. 2436000.+	Phase	Δ m	Helioc. J.D. 2436000.+	Phase	Δ m
574.3934	.4925	0.257	578.3643	.9457	0.342
.3940	.4943	.251	.3652	.9485	.336
.3945	.4959	.251	.3686	.9591	.302
.3959	.5003	.235	.3706	.9654	.300
.3964	.5019	.236	.3713	.9676	.294
.3984	.5081	.221	.3720	.9698	.292
.3989	.5097	.225	.3742	.9767	.268
.3995	.5116	.219	.3750	.9792	.259
.4013	.4172	.206	.3757	.9814	.258
.4020	.5194	.200	.3771	.9858	.251
.4054	.5301	.190	.3780	.9887	.247
.4060	.5320	.189	.3793	.9927	.243
.4066	.5338	.182	.3803	.9959	.232
.4082	.5389	.181	.3827	.0034	.214
.4089	.5411	.174	.3835	.0059	.216
.4111	.5480	.159	.3851	.0109	.202
.4117	.5498	.152	.3857	.0128	.197
574.4134	.5552	+ 0.150	.3891	.0235	.186
			.3901	.0266	.185
578.3110	0.7785	+ 0.421	.3906	.0282	.188
.3130	.7848	.441	.3936	.0376	.180
.3154	.7923	.482	.3949	.0417	.181
.3161	.7945	.501	.3960	.0451	.170
.3187	.8027	.537	.3990	.0545	.170
.3195	.8052	.542	.4000	.0576	.170
.3209	.8096	.562	.4011	.0611	.158
.3215	.8115	.571	.4053	.0743	.148
.3242	.8199	.604	.4088	.0852	.151
.3256	.8243	.624	.4100	.0890	.151
.3269	.8284	.645	.4116	.0940	.153
.3276	.8306	.641	.4140	.1016	.145
.3283	.8328	.652	.4163	.1088	.143
.3317	.8435	.679	.4175	.1125	.147
.3323	.8453	.679	.4184	.1154	.159
.3328	.8469	.682	.4213	.1244	.145
.3359	.8566	.676	.4219	.1263	.148
.3366	.8588	.668	.4247	.1351	.144
.3393	.8673	.645	578.4256	.1379	+ 0.144
.3411	.8729	.629			
.3418	.8751	.625	609.3273	0.0495	+ 0.151
.3427	.8779	.615	.3313	.0620	.153
.3454	.8864	.579	.3342	.0711	.138
.3467	.8905	.557	.3378	.0824	.140
.3475	.8930	.547	.3410	.0925	.140
.3491	.8980	.537	.3443	.1028	.129
.3501	.9012	.523	.3485	.1160	.153
.3507	.9030	.514	.3513	.1248	.160
.3561	.9200	.434	.3542	.1338	.176
.3567	.9219	.425	.3577	.1448	.196
.3574	.9240	.417	609.3613	.1561	+ 0.197
.3589	.9288	.401			
.3597	.9313	.393	1259.3485	0.5964	+ 0.151
.3606	.9341	.382	.3505	.6026	.135
.3613	.9363	.367	.3526	.6092	.147

Table I (continued)

Helioc. J.D. 2437000.+	Phase	Δ m	Helioc. J.D. 2437000.+	Phase	Δ m
259.3540	.6136	0.131	319.3241	.6873	0.218
.3563	.6208	.140	.3264	.6945	.234
.3577	.6252	.145	.3275	.6979	.245
.3598	.6318	.146	.3292	.7033	.249
.3615	.6371	.154	.3305	.7073	.271
.3642	.6456	.158	.3324	.7133	.285
.3655	.6497	.173	.3336	.7171	.306
.3675	.6560	.178	.3354	.7227	.303
.3688	.6600	.178	.3378	.7302	.319
.3709	.6666	.196	.3411	.7406	.331
.3723	.6710	.202	.3435	.7481	.357
.3746	.6782	.210	.3451	.7531	.389
.3760	.6826	.219	.3464	.7572	.396
.3779	.6886	.228	.3488	.7647	.433
.3796	.6939	.236	.3516	.7735	.477
.3834	.7058	.256	.3529	.7776	.474
.3869	.7168	.282	.3548	.7836	.491
.3886	.7221	.289	.3559	.7870	.516
.3910	.7297	.311	.3578	.7930	.543
.3924	.7341	.327	.3590	.7967	.571
.3946	.7410	.349	.3609	.8027	.569
.3961	.7457	.356	.3615	.8046	.591
.3987	.7538	.385	.3641	.8127	.631
.4001	.7582	.403	.3650	.8155	.630
.4022	.7648	.420	.3678	.8243	.670
.4035	.7689	.451	.3701	.8315	.691
.4056	.7755	.468	.3721	.8378	.702
.4071	.7802	.495	.3737	.8428	.704
.4091	.7864	.523	.3745	.8453	.696
.4106	.7911	.550	.3762	.8507	.711
.4124	.7968	.553	.3773	.8541	.699
.4155	.8065	.576	.3789	.8591	.681
.4205	.8222	.640	.3803	.8635	.674
.4237	.8322	.671	.3819	.8685	.651
.4303	.8529	.674	.3830	.8720	.640
.4378	.8764	.605	.3849	.8780	.608
.4402	.8840	.590	.3861	.8817	.587
259.4426	.8915	+ 0.542	.3877	.8867	.575
			.3891	.8911	.552
319.2953	0.5970	+ 0.150	.3907	.8961	.540
.2964	.6004	.140	.3914	.8983	.536
.2983	.6064	.131	.3937	.9056	.493
.2995	.6101	.137	.3950	.9096	.481
.3018	.6173	.146	.3967	.9150	.470
.3028	.6205	.154	.3988	.9215	.435
.3055	.6289	.160	.4011	.9288	.414
.3088	.6393	.180	.4032	.9353	.387
.3097	.6421	.179	.4038	.9372	.366
.3114	.6474	.172	.4073	.9482	.365
.3126	.6512	.162	.4080	.9504	.350
.3144	.6569	.181	.4110	.9598	.327
.3155	.6603	.189	.4125	.9645	.313
.3196	.6732	.196	.4160	.9755	.252
.3214	.6788	.207	.4180	.9818	.252

Table I (continued)

Helioc. J.D. 2437000.+	Phase	Δ m
319.4224	.9956	0.240
.4237	.9996	.224
.4259	.0034	.228
319.4274	.0112	+ 0.223
332.2416	0.1982	+ 0.246
.2439	.2054	.265
.2463	.2129	.270
.2485	.2198	.289
.2507	.2267	.318
.2530	.2339	.323
.2561	.2436	.368
.2584	.2508	.389
.2609	.2587	.419
.2639	.2681	.459
.2669	.2775	.511
.2695	.2857	.555
.2721	.2938	.606
.2744	.3010	.652
.2769	.3089	.710
.2787	.3145	.742
.2810	.3217	.778
.2833	.3289	.796
.2856	.3361	.823
.2883	.3446	.821
.2908	.3525	.818
.2933	.3603	.782
.2958	.3681	.757
.2978	.3744	.726
.3012	.3851	.675
.3038	.3932	.591
.3062	.4007	.566
.3089	.4092	.504
.3114	.4171	.479
.3139	.4249	.449
.3161	.4318	.420
.3187	.4400	.381
.3217	.4494	.341
.3243	.4575	.327
.3269	.4657	.307
.3276	.4679	.303
.3334	.4861	.268
.3388	.5030	.234
.3430	.5162	.210
.3461	.5259	.203
.3487	.5340	.179
.3503	.5390	.182
.3533	.5485	.176
332.3545	.5522	+ 0.181

Helioc. J.D. 2437000.+	Phase	Δ m
343.2683	0.7793	+ 0.489
.2707	.7868	.508
.2717	.7899	.526
.2739	.7968	.559
.2766	.8053	.572
.2787	.8119	.622
.2813	.8200	.664
.2836	.8272	.691
.2861	.8351	.691
.2883	.8420	.688
.2903	.8482	.707
.2924	.8548	.680
.2950	.8630	.662
.2977	.8715	.637
.3005	.8802	.595
.3030	.8881	.567
.3052	.8950	.552
.3070	.9006	.510
.3083	.9047	.489
.3095	.9085	.479
.3113	.9141	.450
.3138	.9219	.436
343.3177	.9342	+ 0.397