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# DISSECTING THE LUMINOSITY FUNCTION OF THE COMA CLUSTER OF GALAXIES USING CANADA-FRANCE-HAWAII TELESCOPE[1] WIDE-FIELD IMAGES

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## ABSTRACT

We determined the relative spatial density of the Coma Cluster galaxies, selected by luminosity and by central brightness, i.e., the luminosity function bivariate in central brightness. The Coma Cluster and control fields were imaged using the CFH12K ($42' \times 28'$) and UH8K ($28' \times 28'$) wide-field cameras at the Canada-France-Hawaii Telescope. Selected *Hubble Space Telescope* (*HST*) images were used for testing. Quantities were derived from measurements in at least two colors, which have the following features: (1) galaxies as faint as 3 times the luminosity of the brightest globular clusters are in the completeness region of our data. (2) We have a complete census (in the explored region) of low surface brightness galaxies with a central surface brightness almost as low as the faintest ones so far cataloged. (3) The explored area is among the largest ever sampled with CCDs at comparable depth for any cluster of galaxies. (4) The error budget includes all sources of errors known to date. Using *HST* images, we also discovered that blends of globular clusters, not resolved into individual components due to seeing, look like dwarf galaxies when observed from the ground and are numerous and bright. When mistaken as extended sources, they increase the steepness of the luminosity function at faint magnitudes. The derived Coma luminosity function is relatively steep ($\alpha = -1.4$) over the 11 magnitudes sampled, but the slope and shape depend on color. A large population of faint low surface brightness galaxies was discovered, representing the largest contributor (in number) to the luminosity function at faint magnitudes. We found a clear progression for a faintening of the luminosity function from high surface brightness galaxies ($\mu \sim 20$ mag arcsec$^{-2}$) to galaxies of very faint central brightness ($\mu \sim 24.5$ mag arcsec$^{-2}$), and some evidence for a steepening. Compact galaxies, usually classified as stars and therefore not included in the luminosity function, are found to be a minor population in Coma.

*Subject headings:* cosmology: observations — galaxies: clusters: individual (Coma) —
galaxies: evolution — galaxies: fundamental parameters — galaxies: star clusters

*On-line material:* machine-readable table

## 1. INTRODUCTION

The luminosity function (LF), i.e., the number density of galaxies having a given luminosity, is critical to many observational and theoretical problems (see, e.g., Binggeli, Sandage, & Tammann 1988). From an observational point of view, the LF is the natural "weight" of all those quantities that need to be weighted against the relative number of objects in each luminosity bin. Furthermore, due to the roles played by flux and surface brightness in the inclusion of objects in any observed sample (faint objects or low surface brightness galaxies are often excluded or underrepresented), the knowledge of the LF and the LF bivariate in surface brightness is fundamental to computing the selection function and is needed to derive the actual galaxy properties from the measured quantities (see, for example, the discussion on the field LF steepness by Sprayberry et al. 1997).

The optical LF of galaxies in clusters has been extensively studied (e.g., to cite just a few papers dealing with a large number of clusters; Gaidos 1997; Valotto et al. 1997; Lumsden et al. 1997; Garilli, Maccagni, & Andreon 1999). However, faint dwarfs and low surface brightness galaxies are outside the reach of most of the previous investigations.

Furthermore, the existence of compact galaxies is usually ignored because, in practice, they are misclassified as stars and then removed from the sample (see, as an exception, Drinkwater et al. 1999). Therefore, an extension of the LF to fainter magnitudes and lower surface brightnesses, without any assumption regarding the compact galaxy contribution and possible bivariate in surface brightness, would be profitable.

Most importantly, the *global* LF hides the true problem (Sandage 1995): the LF is the sum of the LFs of specific types or of any other physically based galaxy classes. In fact, the LF is dependent on the environment, as shown by Binggeli et al. (1988). Maybe the LFs of the morphological types are universal (Binggeli et al. 1988; Jerjen & Tammann 1997; Andreon 1998), but faint galaxies whose morphological types do not fit well in the Hubble (1936) morphological scheme (which has been built for classifying giant galaxies, not dwarfs) raise some concern about the extension of the *type-dependent* LF at faint magnitudes. Galaxies can also be classified on the basis of their central brightness, which also determines where they fall in the fundamental plane (e.g., Bender, Burstein, & Faber 1992), showing that this classification reflects some physical difference between the classes. Therefore, "it would be of great importance to know what the luminosity function looks like when divided into classes of surface brightness" (Kron 1995). In contrast to Hubble

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[1] Based on observations obtained at the Canada-France-Hawaii Telescope and in part at the *Hubble Space Telescope*.

types, classes of central brightness are continuous (as nature often is) and quicker to determine, and can be computed with observations of lower quality than those required to determine morphological types. However, brightness classes merge giant galaxies of different morphological types, which are known to have different properties, into the same class (see, e.g., Andreon 1996 for Coma galaxies).

In this paper we present in three colors the LF and in two colors the LF bivariate in surface brightness of a sample of galaxies in the Coma Cluster of about 1000 members. We measure them down to the magnitude of three bright globular clusters and down to the brightness of the faintest cataloged low surface brightness galaxies. Our studied area is among the largest cluster areas ever observed with CCDs. We use the standard method for computing the LF, namely, the method of differential counts (Zwicky 1957). The method is quite simple: the LF of the cluster galaxies is the difference between galaxy counts in the cluster direction and those counted in a control field direction devoid of (cataloged) clusters. This method has some advantages. (1) It does not require an extensive redshift survey. (2) The redshift dependence of the $K$-correction is not needed. (3) The number of galaxies in each magnitude bin is proportional to the natural frequency with which galaxies are found in the universe, at least in clusters. (4) The difficult problem of calculating the visibility function for a mixed diameter + flux limited survey (as all *field* surveys actually are) is completely skipped, because the cluster sample is naturally volume-limited (details are presented in § 3). The method has the main shortcoming that it applies only to galaxy overdensities and that galaxies in clusters may not be representative of galaxies in general. In that case, the study of cluster galaxies could reveal a correlation between the cluster environment and galaxy properties.

For the Coma Cluster, we adopt a distance modulus of 35.1 mag (i.e., $H_0 = 68$ km s$^{-1}$ Mpc$^{-1}$), according to the direct measure by Baum et al. (1997). The slope of the LF is, as for the Schechter (1976) function, defined by

$$\alpha = -\frac{1}{0.4} \frac{\partial \log \text{LF}}{\partial m} - 1$$

in such as way that a flat (in mag) LF has $\alpha = -1$.

## 2. THE DATA

Observations in $B$, $V$, and $R$ of the Coma Cluster were taken on 1999 January 12 during the CFH12K (Cuillandre et al. 2000; J. C. Cuillandre, in preparation) first light at the Canada-France-Hawaii Telescope (CFHT) prime focus in photometric conditions. Table 1 summarizes a few relevant characteristics of the observations. CFH12K is a 12,288 × 8192 pixel (12k × 8k pixel) CCD mosaic camera, with a 42′ × 28′ field of view and a pixel size of 0″.206. The four dithered images per filter were prereduced (overscan, bias, dark, and flat-field) and then optimally stacked. The CFHT CCD mosaic data reduction package FLIPS (J.-C. Cuillandre, in preparation) was used. Figure 1 shows the studied field. For the present scientific analysis of these very early observations, only the best part of the image is kept (three low-grade CCDs were replaced a few months later), consisting of ∼10.8 CCDs in $V$ and $R$ (1.2 CCDs are of engineering quality) and 8 CCDs in $B$ (1.8 more CCDs are partially vignetted by the only $B$ filter available during these

early observations). After discarding areas noisier than average (gaps between CCDs, borders, regions near bright stars, large galaxies, etc.), the usable area for the Coma Cluster is 0.29 deg$^2$ in $V$ and $R$ and 0.20 deg$^2$ in $B$. Images were calibrated in the Bessel-Cousin-Landolt system through the observation of photometric standard stars listed in Landolt (1992). The scatter of the zero-point measured for the subsample of 7–12 individual stars with large $m$ in the Landolt (1992) catalog (i.e., observed during several nights by him) and in the field of view of the images is ∼0.02–0.03 mag in the three filters. We do not find any trend for a zero-point dependency on magnitude, color, CCD considered, or apparent location in the field of view. Photometric calibration has been cross-checked by using aperture magnitudes of a few galaxies in our field of view listed in de Vaucouleurs & Longo (1988). This external check rules out zero-point errors larger than ∼0.1 mag.

The $B$-band control field is the area around the galaxy NGC 3486 (which occupies less than 10% of the camera field of view, a 10′ × 4′ area). This field shares the photometric calibration of Coma, and we have checked the photometric zero point at a 0.1 mag error level by comparing our aperture photometry of NGC 3486 with that listed in de Vaucouleurs & Longo (1988). Control field (SA 57) images in $V$ and $R$ have been taken from the archive of one of the authors (J.-C. C.). They were taken in 1998 at the same telescope through identical filters, but with the UH8K camera equipped with front-side–illuminated CCDs. UH8K is an 8k × 8k mosaic camera with a 28′ × 28′ field of view and a pixel size of 0″.206. This SA 57 field is centered on a region devoid of (cataloged) clusters and includes a photometric sequence (Majewski et al. 1994), which allows an accurate and straightforward photometric calibration. No significant color term has been detected (as none is present in the CFH12K images). One of the CCDs of the UH8K presented a severe charge-transfer problem, and for simplicity it has been entirely discarded from further analysis. These images cover a large area of ∼650 arcmin$^2$, and they are at an angular distance far enough from the Coma Cluster (a bit more than 2°, corresponding to 3.4 Mpc or a 1.5 Abell radius at the Coma Cluster distance) not to be strongly contaminated by its galaxies, but near enough to sample the overdensity associated with the Coma supercluster. However, our $B$-band control field samples a background several degrees away from the Coma Cluster direction. Field images were processed following the same procedure applied to the Coma Cluster data.

Objects are detected using SExtractor (Bertin & Arnouts 1996), using standard settings (a minimal area of four pixels and a threshold of ∼1.5 $\sigma$ of the sky).

## 3. METHOD OF DIFFERENTIAL COUNTS

The cluster LF (or, equivalently, the relative space density distribution of galaxies of each luminosity) is computed as the difference between galaxy counts in the Coma and in the control field directions (for an introduction on the method, see, e.g., Oemler 1974). The LF bivariate in central brightness is computed in a similar way by subtracting off the contribution due to the foreground and background measured in the control field from counts in the Coma direction.

The method is robust, provided that all sources of error are taken into account. Several of them have already been summarized in Bernstein et al. (1995), Trentham (1997), and
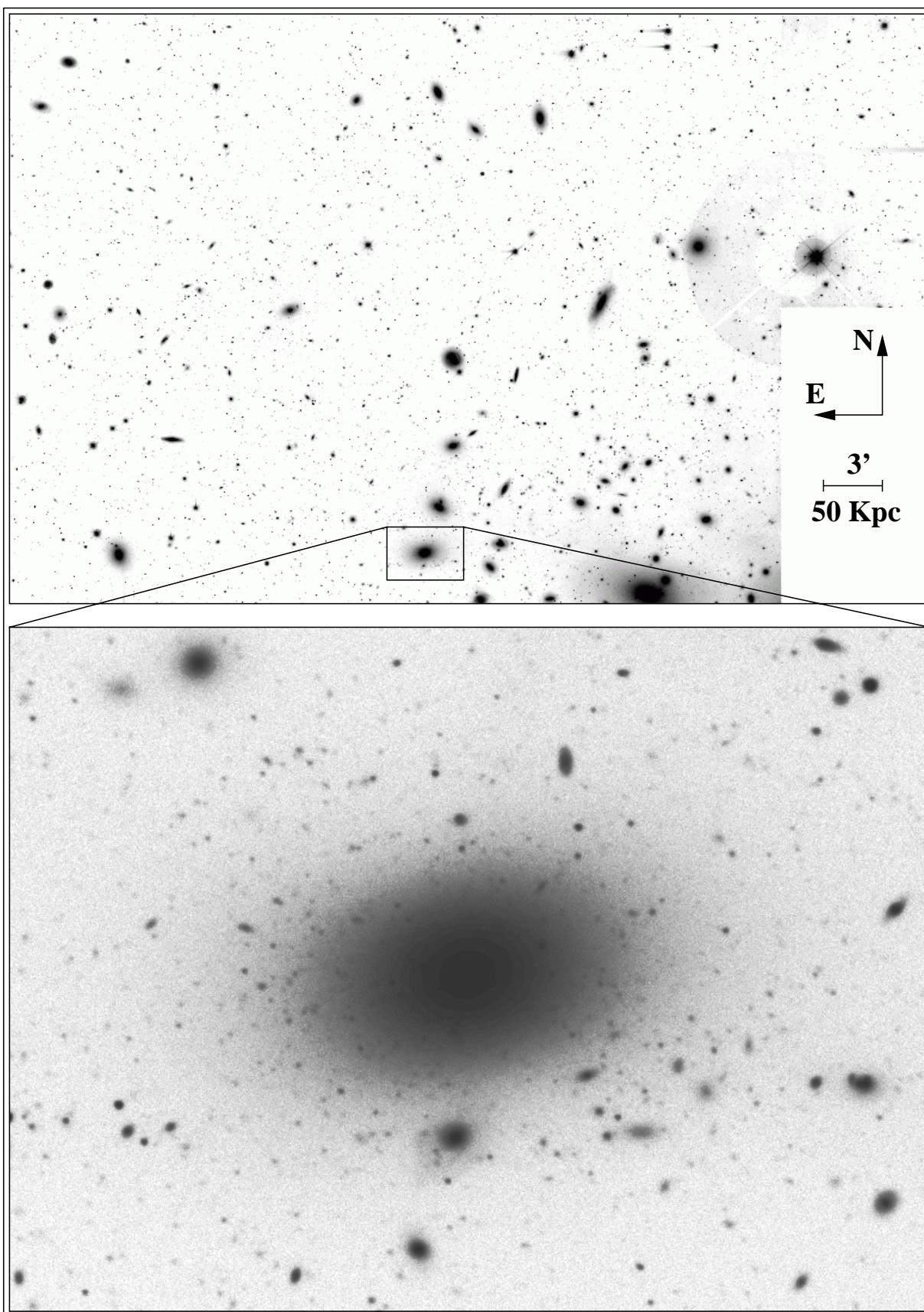
FIG. 1.—*Top*: Whole CFH12K field of view *R* image of the studied field. North is up and east is to the left. The field of view is 42′ × 28′, i.e., 1.2 × 0.8 Mpc² at the Coma distance. Regions with lower quality than average are not considered (such as the bottom right CCD). The studied *B* field includes the central square area. *Bottom*: Galaxy IC 4051, dwarfs, and several GC blends.

TABLE 1
THE SAMPLE

| Pointing | Filter | $t_{exp}$ (s) | Instrument | Seeing (FWHM)[a] (arcsec) | Detection Threshold[a] (mag arcsec$^{-2}$) | Useful Area[b] (deg$^2$) |
|----------|--------|------|------------|---------|---------|---------|
| Coma.............. | B | $180 \times 4$ | CFH12K | 0.88 | 25.0 | 0.20 |
| Coma.............. | V | $180 \times 4$ | CFH12K | 1.23 | 25.5 | 0.29 |
| Coma.............. | R | $120 \times 4$ | CFH12K | 1.04 | 24.5 | 0.29 |
| NGC 3486....... | B | $600 \times 14$ | CFH12K | 0.72 | 26.5 | 0.17 |
| SA 57 ............. | V | $1200 \times 8$ | UH8K | 0.65 | 26.2 | 0.18 |
| SA 57 ............. | R | $1200 \times 8$ | UH8K | 0.65 | 25.7 | 0.18 |

[a] Values are before matching observations to Coma data.
[b] Values are after removing areas noisier than average and halos of bright galaxies.

Driver et al. (1998) and are not repeated here. We point out the following:

1. Extensive simulations show that undetected galaxies cannot be confidently recovered, even statistically, so that completeness corrections are unreliable (Trentham 1997). Therefore, it is preferable to cut the sample, as we did, at the magnitude of the brightest galaxy of the faintest detected surface brightness.

2. Gravitational lensing distorts background counts in the cluster line of sight (Bernstein et al. 1995; Trentham 1997), but it is negligible in very nearby clusters, such as Coma.

This is a quickly growing field, so one should be aware that terms that are currently included in the error budget, such as non-Poisson fluctuations, were not included just a few years ago (but there are exceptions, such as Oemler 1974).

A few sources of concern that should be considered in computing the LF and bivariate LF of galaxies are discussed in more detail in the next sections.

### 3.1. Nearby Background

By using a control field that crosses the Coma supercluster (in $V$ and $R$), we are able to measure the Coma Cluster LF without the contamination of the large-scale structure in which it is embedded, unlike almost all previous CCD determinations of the cluster LF that used control fields in areas too distant from the studied cluster (e.g., Bernstein et al. 1995; Trentham 1998a; Lobo et al. 1997; Biviano et al. 1995). It is easy to show (Paolillo et al. 2001) that a control field too close to the cluster, and therefore contaminated by cluster galaxies, does not alter the shape of the LF, but just changes the LF normalization (and makes the error bars larger), if the LF does not depend too much on the environment.

### 3.2. Photometric Errors

The photometric quality of the night, our checks with the aperture photometry of cataloged galaxies, and the presence of the photometric standard in the field of view all exclude photometric errors as a significant source of error in the determination of the LF or the bivariate LF.

### 3.3. Background Errors

Fluctuations of galaxy counts are surely no longer simply Poisson in nature (i.e., due to only small-number statistics), because of a nonzero correlation function or, in simple words, because of the existence of clusters, groups, and voids. The fluctuation amplitude can be directly measured, as in Bernstein et al. (1995), or estimated by using the Huang et al. (1997) formalism, which uses the galaxy angular correlation function in order to estimate the galaxy count variance averaged over the field of view at a given magnitude and passband. We use this last method, since we have only one control field. Background fluctuations are, in most of the luminosity bins, the largest term in the error budget. Huang et al. (1997) provide the amplitude of the background variance in a given magnitude bin and in a given area, once the characteristic ($M^*$) luminosity of the field population is given, by adopting a galaxy-galaxy spatial correlation function. As a characteristic luminosity, we adopt $B = -20.5$, $V = -21$, and $R = -21.7$ mag (Zucca et al. 1997; Garilli et al. 1999; Paolillo et al. 2001; Blanton et al. 2001). Adopting characteristic luminosities that differ by up to 1 mag does not appreciably change the errors.

Error bars for the bivariate LF further assume (because of the lack of appropriate measures) that the correlation scale of the galaxy angular correlation function is the same for all galaxies, independently of their central brightness. Thus, error bars are approximate, but we verified that a difference in the clustering scale of a factor of 2 produces negligible changes to our results.

Because the LF is the difference between "cluster + background" and "background," the error on the LF has two terms related to the background. In almost all literature LFs, only one term related to the background is taken into account, under the implicit assumption that the "true" background counts are perfectly known.

### 3.4. Adopted Magnitudes and Low Surface Brightness Galaxies

Visual inspection of our images shows that several faint objects in the Coma direction are larger when measured at $\mu \sim 25$ mag arcsec$^{-2}$ than those in our control field, where most of the faint objects are small. The adopted detection thresholds ($\mu = 25.0$, 25.5, and 24.5 mag arcsec$^{-2}$ in $B$, $V$, and $R$, respectively) are fainter than the typical central brightness of low surface brightness galaxies (hereafter LSBGs), which range from 22 to 24 $B$ mag arcsec$^{-2}$ (McGaugh, Schombert, & Bothun 1995; Bothun, Impey, & McGaugh 1997). Quite recently (O'Neil & Bothun 2000), LSBGs with a central brightness as faint as $\mu_B = 24.5$ have been counted.

Therefore, our detection threshold is as low as, or just slightly brighter than, the lowest central brightnesses

sampled so far, with the notable exception of the LSBGs found by Ulmer et al. (1996).

The measured luminosity of LSBGs is strongly dependent on the integration radius because of their shallow surface-brightness profiles. We adopt isophotal magnitudes, recognizing that these magnitudes include a fraction of the object luminosity depending on the object central brightness and on the radial surface-brightness profile. Our magnitudes are not, therefore, *total* magnitudes. In § 5.1 we discuss the impact of this choice on the LF.

Galaxy counts are strongly dependent on the type of magnitude (aperture, isophotal, asymptotic, etc.) used for measuring the flux, and in the cluster direction this effect is exacerbated by nearby (and therefore large) galaxies. Our field counts agree with those in literature (Driver et al. 1994; Trentham 1997) once we select the same type of magnitude adopted in the comparison work. We find that galaxy counts are significantly lower when the adopted isophotal magnitudes are used. We note that galaxies with normal colors are easier to detect in $V$ and $R$ than in $B$, because of the much brighter detection threshold in the latter filter.

### 3.5. *Completeness*

Since undetected LSBGs cannot be recovered, we need to cut the sample at the magnitude of the brightest LSBGs of the faintest detectable central surface brightness. A detailed explanation of this method is described in Garilli et al. (1999). By definition, the sample will be complete down to the cutting magnitude. For our sample, the cutting magnitudes are $R = 23.25$, $V = 23.75$, and $B = 22.5$ mag. At these magnitudes, the measured signal-to-noise ratio (S/N) is about 20.

#### 3.5.1. *LSBGs*

Because of the low surface brightness threshold, LSBGs are included in our catalog. Galaxies with extremely low central surface brightness ($\mu_0 \gtrsim 25$ mag arcsec$^{-2}$) are correctly excluded in our LFs because their magnitude at the chosen isophote is exactly zero.

#### 3.5.2. *Eddington Bias*

Catalogs suffer a typical incompleteness: because of the noise, galaxies can be undetected even if their central brightness is slightly brighter than the threshold and can be detected even if their brightness is below the threshold. Furthermore, the noise and the increasing galaxy counts at faint magnitudes include a larger number of galaxies in catalogs than they exclude (this effect is called Eddington bias). By keeping only high-quality data, as we do by cutting the samples at the completeness magnitude, incompleteness and Eddington bias are minor concerns. For example, in our fainter bin, the observed minimal S/N is ∼20 in $R$, while at the faintest magnitude and at the faintest surface brightness, the observed S/N of the central brightness is ∼10 in $R$.

### 3.6. *Image Properties Matching*

The control-field images are deeper than Coma images and taken under better seeing conditions, with the exception of the $B$ images that were taken during similar seeing conditions (see Table 1). In order to compute the LF and the bivariate LF, it is necessary to match the properties of the control and program images. First of all, we match the seeing profile, convolving control-field images with an appro-

priate kernel. The match of the point-spread functions is checked by verifying that stars lay on the same mag versus central brightness locus in both the Coma and the control-field images. Then, the noise in the images is matched by adding Poisson noise. We checked that the noise matching is not crucial, i.e., that the results do not change by more than the error bars. This holds because we take the general approach of completely discarding all data that are affected by noise. By cutting our sample to a minimal S/N of 20, noise is not a concern.

#### 3.6.1. *Star/Galaxy Classification and Compact Galaxies*

Careful numerical simulations performed by us show that existing elliptical galaxies as compact as NGC 4486B or M32 could not be recognized as galaxies in our images *independent of their luminosity* if they were in the Coma Cluster,[2] and they look like stars on our images.

As previously stated, the LF is given by the difference of *galaxy* counts. What is actually usually taken in literature is the difference of counts of *extended* objects. The two calculations give the same result when galaxies and "extended-object" classes perfectly overlap; however, this hypothesis is not satisfied, even in a cluster as near as Coma.

If compact galaxies were excluded ab initio, then they would not be counted in the LF.

How to solve this problem? In two ways, depending on the object luminosity:

*Bright objects.*—Our control field is close enough in the sky to the Coma Cluster to assume that star counts are equal, within the statistical fluctuations, in the two pointings (which are both at the Galactic pole and whose nearest corners are less than 1° apart). We verified by means of Besancon models[3] that the variation of star counts due to the small differences in Galactic latitude and longitude between Coma and the control field is negligible (far less than 1%). We can check the existence of bright compact galaxies (misclassified as stars) by simply comparing the number of the starlike objects in the Coma and control field directions. In the control field there are 236 objects brighter than $V = 20.5$ mag classified as stars. The expected number of stars in the Coma pointing (which covers a larger area) is thus 384. We found 382 stars, two less than the expected number, and therefore no excess of compact objects in the Coma direction is found. The 1 $\sigma$ upper limit to the number of compact ellipticals in the studied portion of Coma is 25. Even if these 25 galaxies were present (while we found $-2$ galaxies), they would be a minor population (9% of the net number of Coma galaxies brighter than $V = 20.5$ mag), and they would change the measured Coma Cluster FL by less than the error bars.

Because of the verified paucity of compact galaxies in Coma, bright stars (brighter than $V = 20.5$ mag) are individually removed from galaxy counts. Unlike previous works, we have verified that compact galaxies are a minority population before discarding them.

*Faint objects.*—At faint magnitudes, even not-so-compact galaxies can be misclassified as stars because of noise in $V$ and $R$. In fact, we found that several objects from the con-

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[2] This concern has been raised by Dave Burstein, whom we warmly thank.

[3] See http://www.obs-besancon.fr/www/modele/modele.html.

trol field are misclassified at $V > 21$ mag, when the images are degraded to match the seeing and noise of Coma images. Furthermore, star counts differ in the Coma and field directions at faint magnitudes (but not at bright magnitudes), whereas they should be equal according to the model. Therefore, stars are not individually identified and removed, but statistically subtracted, and starlike objects in the Coma direction due to compact galaxies are not thrown away during the star/galaxy classification. As a consequence, the problem of the star/galaxy misclassification (due to both object faintness and intrinsic object compactness) is overcome. This way, the problem represented by compact objects is solved, but at the price of larger error bars because of the statistical subtraction. We stress that *measured* star counts are used, not the expected ones.

### 3.7. *Globular Clusters and Their Blends*

Even a casual inspection of the region around IC 4051, an early-type galaxy in the studied field shown in the bottom panel of Figure 1, shows a huge population of extended sources clustered around this galaxy. Other extended sources are present near NGC 4481, another bright Coma elliptical in our field of view. These objects are extended and as bright as $R = 21$ mag. Since globular clusters (hereafter GCs) of IC 4051 have a turnoff magnitude of $V \sim 25$ (Baum et al. 1997) and are unresolved at the Coma distance (i.e., they are point sources), this huge population cannot be formed by individual GCs. In order to understand how many extended sources there are at each magnitude, we compute their luminosity function. We first subtract a model of the galaxy, obtained by fitting its isophotes. Then, we compute the counts in an annulus centered on IC 4051 of $6''$ and $31''$ of inner and outer radii, respectively, and in a control region of the same area $160''$ east of IC 4151. In the annulus on IC 4051, we found an excess of $2.3 \times 10^5$ extended objects per mag per deg$^2$ at $R \sim 24$, with respect to the control field. The number of extended objects in the annulus is 4 times larger than in the control field, and the

excess is statistically significant, even including non-Poisson fluctuations. The luminosity function of these extended sources has a slope, in a 3 mag range fainter than $R = 21$ mag, compatible with the slope of the GCs specific frequency (0.4). The brightest of these extended sources has $R = 21$ mag, i.e., they are $\sim 6$ mag brighter than the GC turnoff (directly measured by Baum et al. 1997 for this galaxy) and 3.4 mag brighter than the tip of the GC population (which in turn is ill defined, because the number of bright GCs decreases exponentially at bright magnitude without any clear break).

Since IC 4051 has been observed by the *Hubble Space Telescope* (*HST*) (Baum et al. 1997), we can use the superior angular resolution of the *HST* for better understanding these sources. *HST* archive images of IC 4051 have been retrieved, and the galaxy has been modeled and subtracted off, as for the ground images. Figure 2 shows the residual image of IC 4051, as seen in our ground image (*left panel*) and from space (*right panel*). In the left panel, the actual galaxies revealed by the *HST* are marked by circles. Note that only one faint object is circled. All the other objects are blends of a few point sources (typically three to five), unblended at the *HST* resolution. Most of them are brighter than our completeness limit (and the limits of other deep probes of the LF), and therefore would be counted as galaxies in the LF. The two brightest blends in the *HST* field of view have $R = 21.3$ and $20.8$ mag. The large majority of *HST* point sources are GCs (Baum et al. 1997), and therefore the large majority of our extended sources are blends of GCs. However, a few extended sources could be blends of any type of point sources, such as foreground stars, GCs, and groups of GCs if they exist, because even *HST* cannot individually distinguish GCs at the Coma distance from foreground stars. In particular, the two brightest sources, marked with diamonds in the *HST* image, are largely dominated by a bright single point source, quite bright for a single GC, whose identification as a GC or foreground star is possible only on a statistical basis.
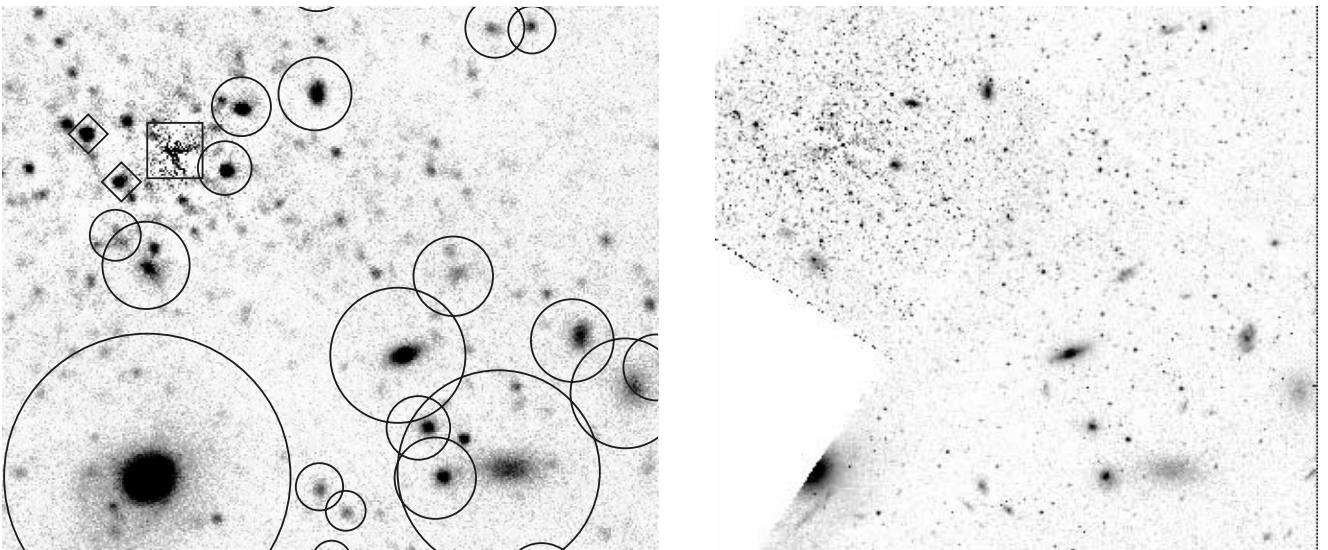


Fig. 2.—Residual $R$ image after having subtracted a model of IC 4051 from the original image. The left panel shows a part of the ground image, while the right one displays a part of the *HST* image. The galaxy, shown in the bottom panel of Fig. 1, is much larger than the field of view of this cutout of $68''$ angular size. In the left panel, the square marks IC 4051's center, and circles the true galaxies in the field, as confirmed by the *HST* image. Most of the remaining objects, most of which look like extended sources from the ground and are brighter than the completeness magnitude, are unresolved blends of GCs. Diamonds mark the two brightest blends in the *HST* field of view, having $R \sim 21$ mag.

Simple statistical arguments on the luminosity function of GCs suggest that the very brightest of our blends are blends of GCs and some other source unresolved at the *HST* resolution (including groups of GCs if they exist), while the other ones are instead, in large majority, blends of GCs alone.

Inspection of the *HST* image of another large galaxy in our field, NGC 4481 (Baum et al. 1995), confirms our findings for such blends.

To summarize, our large population of extended sources is, in large majority, blends of GCs. While GCs are point sources, their blends are a source of concern because they have the unfortunate property of being classified as single extended sources in typical seeing conditions, and thus are included in the galaxy counts. Being blends of a few/several GCs, these sources are brighter on average than GCs. Therefore, GC blends not only affect typical GC magnitudes ($V \sim 27$ mag), but also bias much brighter counts (as bright as $R = 21.5$ mag), and are thus pernicious because they are extended sources. Their density is high near giant ellipticals: 4 times higher than galaxy counts in the considered region of IC 4051.

Previous works studying the deepest part of the galaxy LF may be affected by GC blends at faint magnitudes. For example, the determination of Bernstein et al. (1995) of the Coma LF at very faint magnitudes, measured in the NGC 4874 outer halo, optimistically assumes that the GCs contamination starts at $R = 23.5$ mag (while it starts at 2 mag brighter) and rules out a GC contamination at brighter magnitudes because their objects are marginally resolved, while we found that GC blends also share this property. De Propris et al. (1995) found a steep LF over their very small studied field (with a slope that nicely corresponds to those of our GC blends), and they correctly warn the reader of the possible contamination of their galaxy counts by an unusual population of GCs. Actually, we believe that their counts are contaminated by GC blends more than by an unusual population of GCs, because of the similarity of the properties of their possible unusual population of GCs to our GC blends and because Trentham (1998c) does not find such a steep slope when observing one of the clusters of de Propris et al. (1995) over a larger field of view (where the contribution of GC blends is washed out).

Thus, the points of published LFs at $M \gtrsim -14$ mag should be regarded with caution as long as the area surveyed is comparable to (or smaller than) that occupied by bright galaxies. Because of this potential source of error, we generously mask out areas to discard a few bright galaxies with a large GC population and a halo. Residual unflagged contamination is diluted by the very large field of view of our images. Flagging areas occupied by large galaxies also solves in the simplest way the problem of crowding, because the unflagged area is mostly uncrowded.

## 4. THE COMA CLUSTER LF AND THE BIVARIATE LF

With respect to previous LF determinations, our work presents new features:

1. The control field, although only a single one, is at an ideal angular distance from the cluster pointing: far enough from the Coma Cluster to not be strongly contaminated by its galaxies, but near enough to correctly sample the density enhancement of the Great Wall (in which the Coma Cluster is embedded). Even if the control field were contaminated by Coma Cluster galaxies, the shape of the LF would not be altered by this contamination. Background fluctuations are included in the error budget.

2. Compact galaxies are not lost in the star/galaxy classification, and no assumption about their existence, or contribution to the LF, is made.

3. We do not assume that galaxy counts in the control field are the "true" average errorless background, and in measuring error bars, we count background errors twice.

4. Blends of GCs are not counted in galaxy counts.

### 4.1. *Luminosity Function*

Figure 3 (*filled circles*) shows the Coma Cluster LF down to $R = 23.25$, $V = 23.75$, and $B = 22.5$ mag. Note the large number of galaxies per magnitude bin in our $R$ and $V$ LFs and the absolute faintness of studied galaxies ($M_R \sim -11.75$, $M_V \sim -11.25$, and $M_B \sim -13$ mag), whose luminosity exceeds the tip of the GC LF ($M_V \sim -10$ mag) by less than a factor of 3 in flux (in the deepest bands). The LF extends over an 11 mag range, and it is one of the deepest ever derived from CCD photometry.

The LFs in the three filters present both similarities and differences. The LFs seem truncated at the bright end ($R = 12$, $V = 13.5$, $B = 15$ mag). This abrupt truncation is due to the fact that all galaxies brighter than the first plotted point are removed from the sample because of their potential large population of GCs (and their blends).

At intermediate luminosities ($B < 18$, $V < 16$, and $R < 16$ mag), the LFs are fairly flat.

At fainter magnitudes, the LFs are steep in $R$ and $V$, and with a much shallower slope in $B$. Of course, the exact slope depends on the considered magnitude and filter and can be precisely computed by the reader at his favorite magnitude by taking the best-fit functions, whose parameters are listed in Table 2,[4] or by using the tabulated LF of Table 3. The typical slopes range from $-1.25$ in $B$ to $-1.4$ in $R$ and $V$. In the three filters, we do not see any clear turnoff of the LF, meaning that galaxies can be as faint as three very bright GCs, and such galaxies are the most numerous in the studied Coma region. In the $V$- and $R$-bands, there is a hint of a flattening of the LF at faint magnitudes, but the statistical evidence for it, or for a turnoff of the LF, is small due to the large errors.

The quality of our LFs decreases going toward blue filters for two reasons: first of all, the surveyed area in $B$ is 30% smaller than in $V$ or $R$. Second, bluer filters preferentially select blue galaxies, abundant in the field and rare in clusters, and therefore the contrast between members and interlopers is low. Because of these reasons, the bivariate LF in the $B$-band is not presented.

In the $R$-band, the LF shape is not well described by the Schechter (1976) law, because their best fit has $\chi^2 \sim 37$ for 18 degrees of freedom. A function with more free parameters better describes the data. The best fit with a third-order power law (i.e., with one more free parameter) is overplot-

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[4] There are shortcomings in inferring a slope from a parametric fit to the data (see, e.g., Merritt & Tremblay 1994) when the term "slope" indicates the local derivative of the underlining function in a given point. However, the latter is not the usual meaning given to the term "slope," which instead means the typical change of the LF over a finite magnitude range, as the Schechter and our power-law functions provide.
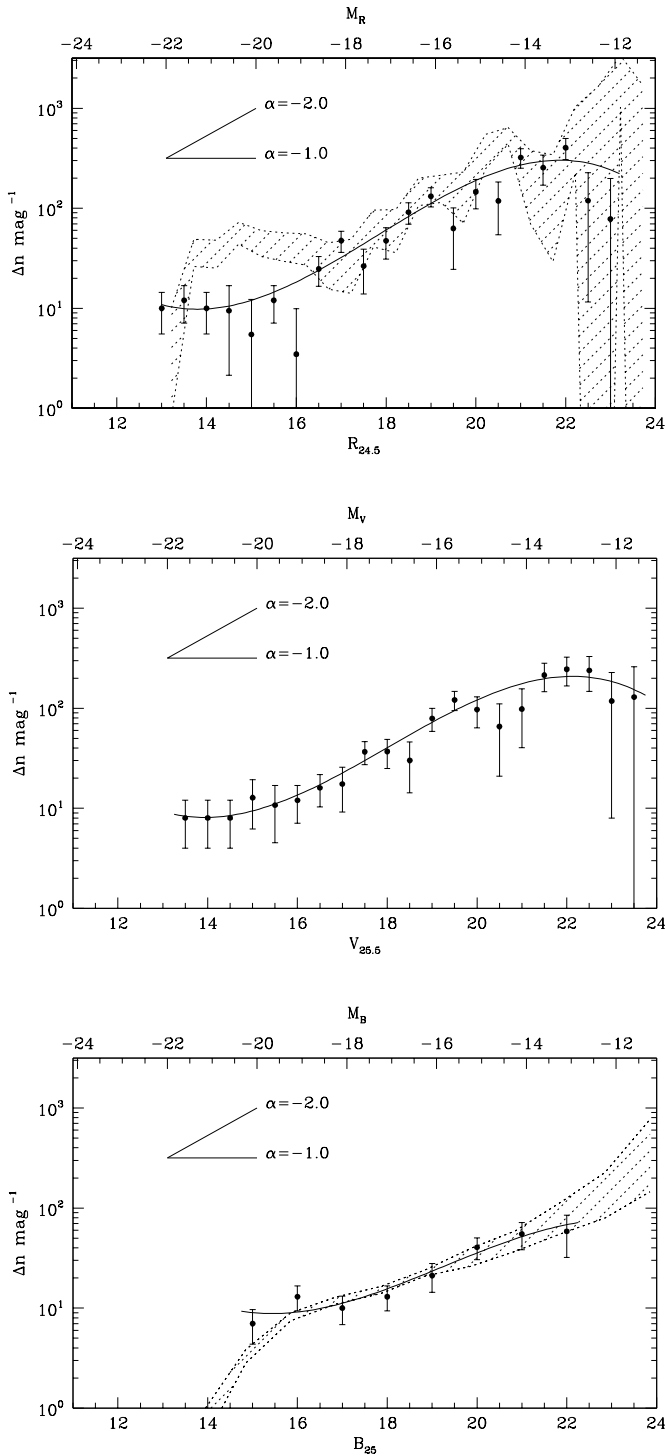
FIG. 3.—Luminosity function of the Coma Cluster in the *R*-, *V*-, and *B*-bands (*filled circles*). Both apparent and absolute magnitudes are shown on the abscissa. Error bars take full account of Poisson and non-Poisson errors, i.e., they include errors due to the presence of under/overdensities along the lines of sight. The thick curve is the best fit to the data with a three-degree power law. In the *R* (*top*) panel, the hashed region delimits the LF determined by Trentham (1998a). In the *B* (*bottom*) panel, the hashed region delimits the composite *B* LF, averaged over almost all the LFs from literature (from Trentham 1998b).

ted in Figure 3 (*solid curves*). The best-fit parameters are listed in Table 2. The reduced $\chi_\nu^2$ is ∼1, suggesting a good fit.

The LF does not continue to steepen any more at longer wave bands, because in the *H*-band ($\lambda \sim 1.6 \, \mu$m), the LF of the same portion of the cluster has slope $\alpha = -1.3$ down to

$H = 18.5$ mag (Andreon & Pelló 2000), which roughly corresponds to $R \sim 21$ mag or $M_R \sim -14$ mag.

At $R \sim 16$ mag there is a hint of a possible dip in the LF: approximately five Coma galaxies are expected in the half-magnitude bin, while approximately 0.7 are observed. However, the statistical significance of the effect is negligible (∼1 $\sigma$). This feature is common among the Coma LFs determined so far: it has been found in the photographic *V*-(Godwin & Peach 1977) and *b*-band (Biviano et al. 1995), and in the near-infrared *H*-band (Andreon & Pelló 2000).

### 4.2. *Comparison with the Literature*

The shaded regions in Figure 3 delimit the best previous determinations of the LF. In the *R*-band, the shaded region is the LF of the "deepest and most detailed survey covering...a large area" (Trentham 1998a). Trentham (1998a) surveyed a ∼0.18 deg² area of the Coma Cluster, i.e., a 40% smaller area than the present survey, overlapping but not coincident with the Coma Cluster region studied in this paper. At $R > 21$ mag ($M_R > -14.5$ mag), the *literature* LF is quite noisy and does not constrain the LF. The bright part of the Trentham *R* LF disagrees with those computed from surveys of a large number of clusters (e.g., Paolillo et al. 2001; Piranomonte et al. 2001), while our LFs are truncated because we removed giant galaxies and their surrounding areas where the GC blend contamination was potentially high. The two LFs are normalized to $R \sim 18$ mag and show reasonable agreement, given the errors, in the region of validity of both LFs. We note that error bars in Trentham (1998a) are, in our opinion, underestimated because they count background fluctuations only once, instead of twice as we advocate. The much shallower *R*-band LF of the Coma Cluster computed by Secker & Harris (1996) shows a similar agreement.

In the *V*-band, no LF comparable in depth and extension to the present one is known to the authors.

In the *B*-band, Trentham (1998b) summarizes our present knowledge on the LF by computing the composite cluster LF, averaging over almost all literature LFs based on wide-field deep images, including Virgo (Sandage, Binggeli, & Tammann 1985), for example. The shaded region in the bottom panel of Figure 3 shows his result, once data are sampled at 1 mag bins (which help in reducing the scatter) and vertically shifted to match our points at $B > 16$ mag. Our data agree well with the Trentham (1998b) composite LFs, and the agreement should increase if Trentham's (1998b) error bars are made larger in order to include the background fluctuations twice in the error budget.

To summarize, we compute the Coma LF in three bands, over a very large magnitude range (up to 11 mag) with good statistics. Our results agree with previous LF determinations on the common magnitude range. The discussion of the LF is deferred until after the presentation of the bivariate LF.

### 4.3. *The Bivariate Luminosity Function*

The quality of our LF determination allows a truly new interesting quantity to be accurately determined: the bivariate LF, i.e., the LF of galaxies of a given central brightness. Central brightness is measured on the images (which are convolved by the seeing disk, whose FWHM correspond to ∼1 kpc at the Coma Cluster distance) in a 0.25 kpc aperture. At the time of the submission of this paper, this determina-

TABLE 2
BEST FITS

| Luminosity Function | Range | Zero-Order Coefficient | First-Order Coefficient | Second-Order Coefficient | Third-Order Coefficient | $\chi^2_\nu$ |
|---|---|---|---|---|---|---|
| LF-$R$........................... | 13.00–23.25 | 28.66 | −5.1346 | 0.30346 | −0.005672398 | 1.3 |
| LF-$V$........................... | 13.25–23.75 | 26.73 | −4.7455 | 0.27751 | −0.005130364 | 0.7 |
| LF-$B$........................... | 14.75–22.25 | 27.76 | −4.4728 | 0.24188 | −0.004187621 | 1.0 |
| LF-$R$, $\mu < 19$ ................ | 12.75–17.75 | 1.80 | −0.08 | . . . | . . . | . . . |
| LF-$R$, $\mu = 19.5$ ........... | 15.5–19.0 | −2.13 | 0.17 | . . . | . . . | . . . |
| LF-$R$, $\mu = 20.5$ ........... | 16.5–20.0 | −4.72 | 0.33 | . . . | . . . | . . . |
| LF-$R$, $\mu = 21.5$ ........... | 17.5–21.5 | −5.92 | 0.38 | . . . | . . . | . . . |
| LF-$R$, $\mu = 22.5$ ........... | 18.5–22.25 | −7.21 | 0.43 | . . . | . . . | . . . |
| LF-$R$, $\mu = 23.5$ ........... | 19.5–23.0 | −18.83 | 0.48 | . . . | . . . | . . . |

NOTE.—The quoted coefficients are a simple empirical description of the LF shape and the large number of digits should not be taken as an indication of the good quality of the fit. Furthermore, the fit should not be extrapolated outside the quoted range of validity.

tion was the first so far accurately computed for any environment, to our best knowledge. The previous larger effort in this direction is presented in de Jong (1996), with a study of a sample of 86 field galaxies (while our sample includes ∼1000 cluster member galaxies), whose bivariate LF is "more of qualitative than quantitative interest" (de Jong 1996). After this paper was submitted, two more bivariate LFs (Cross et al. 2001; Blanton et al. 2001) were submitted for publication.

Figure 4 shows a three-dimensional view of the $R$ bivariate LF. Two-dimensional views, at fixed surface brightnesses, are presented in Figure 5 for both $V$ and $R$ filters. On the latter figures, error bars can be plotted, and therefore the quality of the bivariate LF can be appreciated. Brightness bins are 1 mag arcsec$^{-2}$ wide, except the brightest one, which is wider to improve the statistic. Reducing the amplitude of the first brightness bin slightly decreases the statistics but does not significantly change the results.

The way brightness is measured limits the luminosity range accessible to a galaxy of a given brightness: since the central brightness is measured on a finite area, objects of a given central brightness have a minimal flux. The hashed regions in Figure 5 mark the regions that could not be occupied by our objects because of such a minimal flux. Furthermore, no object of a given central brightness could be fainter than a star of the same central brightness. An arrow in the plots marks this magnitude. In this diagram, compact galaxies fall below the arrow. Therefore, the available observational range for each bivariate LF goes from −∞ to the arrow, including these limits. Therefore, in Figure 4 the region on the right not occupied by galaxies is empty because of the rarity of such types of galaxies, whereas the empty region on the left is devoid of galaxies because of the way surface brightness is measured.

At all brightness bins, galaxies occupy a bounded range in luminosity smaller than the whole available range. Although a distribution with a finite width is expected, we can now quantify it. The plotted values shown in Figures 4 and 5 are tabulated in Table 3.

Galaxies of very large size or very flat surface brightness profile (i.e., near the left end of each bivariate LF plot) are

uncommon. In fact, galaxies with $m < \mu - 4$, i.e., more than 4 mag brighter than their central brightnesses, are almost absent in our sample. Furthermore, the bright end moves toward fainter magnitudes when the central brightness decreases.

At the faint end, the LFs seem to flatten or turn down. Furthermore, the point at the arrow magnitude, i.e., where objects are as compact as the seeing disk, is seldom on the extrapolation of points at brighter magnitudes. While the possible flattening at magnitudes slightly brighter than that of the arrow is uncertain, the points below the arrow are systematically lower than brighter points. The rarity of galaxies at the magnitude marked by the arrow, when compared to the expected value based on the trend at brighter magnitudes, is not due to the fact that compact galaxies are removed in the star/galaxy classification or implicitly not supposed to exist, because our derivation of the LF does not follow this path, unlike previous works. The found rarity of compact galaxies means that most of the galaxies at the Coma distance are extended sources at our resolution, and this explains why our LF, which also counts compact gal-
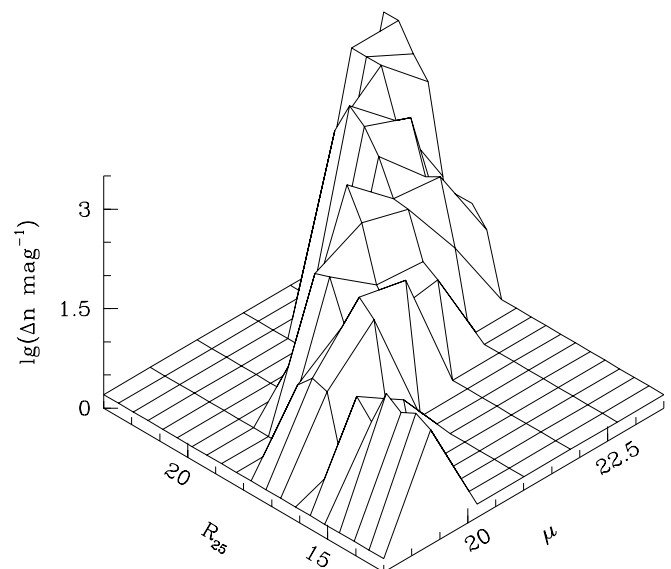
TABLE 3
THE LUMINOSITY FUNCTIONS

This table is available only on-line as a machine-readable table.



FIG. 4.—Three-dimensional view of the bivariate LF of Coma galaxies in the $R$-band. The empty region on the left is devoid of galaxies because of the way brightness is defined, while the region on the right is empty because of the rarity of such a type of galaxy in Coma.
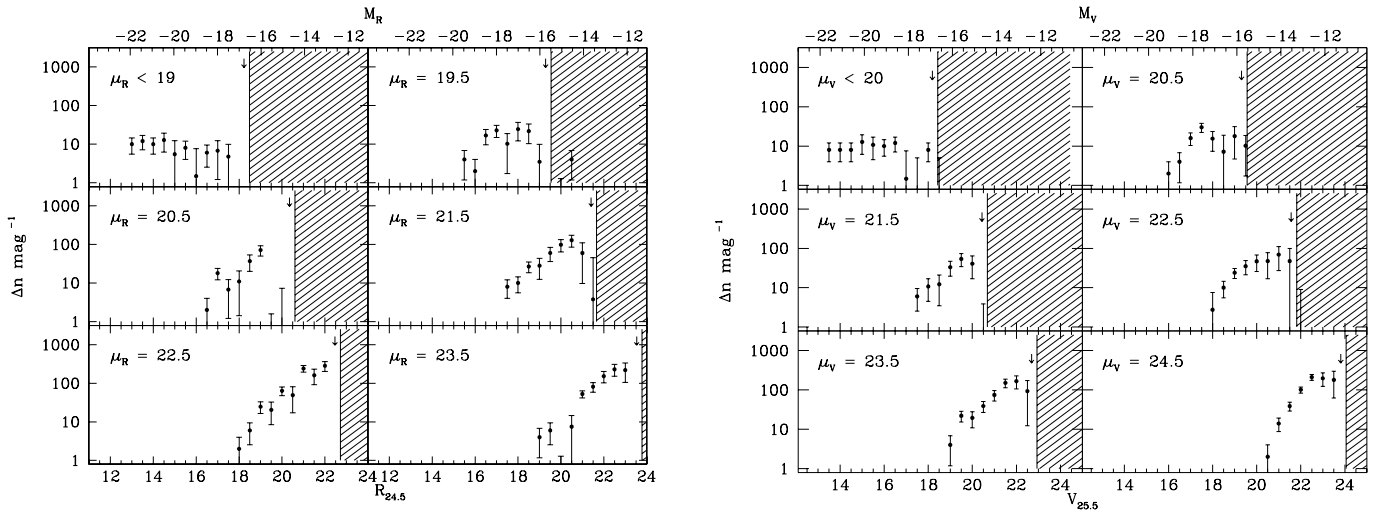
Fig. 5.—Bivariate LF of Coma galaxies in the *R*- and *V*-bands. There is a clear progression from flat and bright LFs of high surface brightness galaxies to steep and faint LFs of LSBGs. Error bars are as in Fig. 3. The seeing and sampled area for the brightness determination make the hashed regions of the diagram forbidden to galaxies.

axies, agrees with previous works that instead implicitly assume that compact galaxies do not exist.

Lacking a *field* bivariate LF,[5] it is difficult to say whether the Coma Cluster is effective in harassing LSBGs, as advocated by Moore et al. (1996, 1999), or whether the bivariate LF is the same in the two environments and tells us more about galaxy formation and evolution in general.

Figure 6 presents the *R*-band LF (*curved line*) and a linear fit to the LF of the galaxies of each central brightness (*straight lines*). The best-fit parameters are listed in Table 2. During the fit process, we manually discarded outlying points, and we arbitrarily adopted a linear (in log units) fitting function. For some LFs, a higher order function would be preferable, but at the price of overfitting most of the other LFs. It is quite apparent that LSBGs dominate the LF at faint magnitudes, while high surface brightness galaxies dominate the bright end. High surface brightness galaxies ($\mu_0 < 20.0$ mag arcsec$^{-2}$) have a shallow LF ($\alpha \sim -1$), while LSBGs have a steep and fainter LF (see also Fig. 5). There is a clear trend for a faintening of the LF going from high surface brightness galaxies to faint and very faint central brightnesses, which is also directly visible in the data in Figure 5. There is also some evidence for a steepening of the LF, in particular when galaxies of high surface brightness are considered. The trend is still there, even when not considering at all the LF of galaxies with $\mu_R < 19.0$ (or $\mu_V < 20.0$) mag arcsec$^{-2}$, which is determined on a wider brightness range than the other bivariate LFs, or when adopting for this bin a 1 mag arcsec$^{-2}$ width, as we did for the other brightness bins. Surface brightness is correlated to luminosity, since galaxies of lower central surface brightness often have fainter magnitudes (Fig. 4), as typically found in incomplete volume samples (e.g., van der Hulst et al. 1993; de Blok, van der Hulst, & Bothun 1995; Impey & Bothun 1997; van den Hoek et al. 2000).

## 5. DISCUSSION

### 5.1. *What Would Likely Change by Going Deeper?*

First of all, we emphasize that adopting a total magnitude instead of an isophotal magnitude should probably slightly modify the LF and the bivariate LF by shifting them to the left and making them steeper. The LF becomes slightly brighter, i.e., moves to the left, because a fraction of the total flux is below the brightness threshold; thus, the adopted isophotal magnitudes underestimate the total flux of the galaxies. For galaxies of high central brightness (that are bright, as described in § 4.3), the fraction of lost flux is fairly small (Trentham 1997) if their surface brightness profile below the observed brightness threshold follows the extrapolation of the observed part: the galaxy flux has already been integrated over a wide brightness range, and the flux below the threshold is negligible. On the other hand, the fraction of the total flux below the brightness threshold
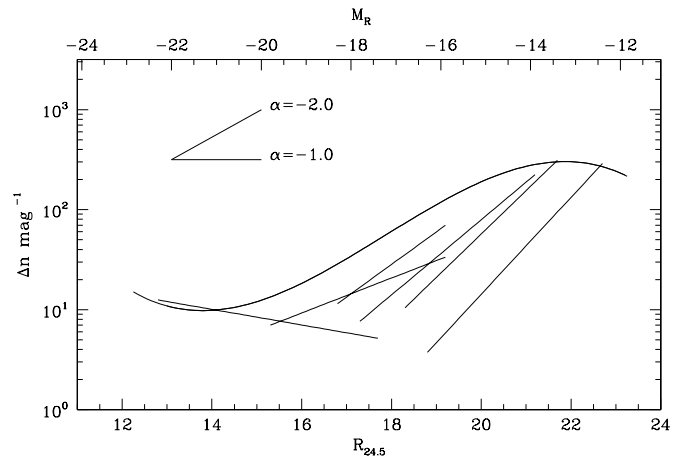


Fig. 6.—Dissection of the *R* LF in central brightness. The curved line is the Coma *R*-band LF, while the straight lines are the contribution to the LF of the galaxies of each central brightness. Galaxies of large central brightness have a flat LF (on the left part of the graph), while LSBGs have a steep LF (on the right). Bins of central brightness are less than 19, 19.5, 20.5, 21.5, 22.5, and 23.5 mag arcsec$^{-2}$.

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[5] During the revision of this paper, two bivariate *field* LFs appear. Nevertheless, no comparison with these works can be performed: the 2dF (Cross et al. 2001) and SLOAN (Blanton et al. 2001) bivariate LFs use different definitions of brightness than we have adopted, and furthermore, 2dF adopts an indirect measure of the galaxy brightness.

increases going toward faint magnitude objects because these objects often have faint central brightnesses (§ 4.3), and thus their isophotal magnitude is integrated on smaller and smaller brightness ranges. Assuming that galaxies have perfect exponential surface brightness profiles, we find that the corrections range from $-0.75$ mag for the galaxies of lowest surface brightness to $-0.05$ mag for high surface brightness galaxies. Because of this correction, the $R$-band LF changes its slope $\alpha$ by $-0.05$, i.e., by 3%. The LF likely becomes steeper when adopting total magnitudes for yet another reason: galaxies whose central brightnesses are below the present brightness threshold will be counted, and they are likely preferably faint, if the trend presented in Figure 5 continues at lower surface brightnesses and magnitudes.

### 5.2. *Large Numbers of LSBGs in High-Density Regions*

We found a large quantity of LSBGs in the core of the Coma Cluster. The cluster environment is often regarded as hostile to the formation and survival of LSBGs; in fact, as much as 90% of the stars in LSBGs can be harassed from them (Moore et al. 1999). On the other hand, the harassment process may contribute to the production of LSBGs in clusters (Moore et al. 1996). Therefore, the " harassment " paradigm has no predictive power on the number of LSBGs in clusters. Maybe the cluster LF might tell us about cluster-related processes in too detailed a level for a prediction with the present-day models.

Phillipps et al. (1998a) examine the dissimilarity of the dwarf population in different environments. Their faintest dwarfs are 5 mag brighter than our limit, i.e., they are talking about normal dwarfs, not faint ones. They note a variation in the LF shape that is driven in part by galaxy density: at low galaxy densities, both steep and shallow LFs are permitted, while at high galaxy densities, only flat LFs are observed. We computed, according to the recipes of Phillips et al. (1998a), the giant-to-dwarf ratio (which in our case used only galaxies with S/N greater than $\sim$300) and found a giant-to-dwarf ratio of $\sim$7 $\pm$ 1, at the projected galaxy density of 26 galaxies Mpc$^{-2}$. The result is near the extrapolation of the outer envelope of their proposed correlation. This calculation is computed using $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$ for consistency with Phillips et al. (1998a).

### 5.3. *Missed Galaxies in the Field LF Determination?*

There is ample discussion in the literature whether or not the local field LF is well determined at faint magnitudes, because most of the surveys purporting to be magnitude-limited do not take surface brightness effects into account (Disney 1976; Sprayberry et al. 1997; Phillipps et al. 1998b). Optical surveys reveal an excess of faint blue galaxies over and above the number predicted by simple models relating local to distant observations (e.g., Tyson 1988; Lilly, Cowie, & Gardner 1991). The flat faint-end slope measured in the local $B$-band LF of galaxies (Efstathiou, Ellis, & Peterson 1988; Loveday et al. 1992) plays an important role in this interpretation, since the faint blue galaxies might otherwise be explained by a local population of intrinsically faint (and nearby) galaxies (Driver & Phillipps 1996).

Recent surveys, such as the cluster surveys by Impey, Bothun, & Malin (1988) and Irwin et al. (1990) and the field survey by Impey et al. (1996) have taken this potential source of bias into account by deliberately searching for

LSBGs. However, their search is limited to giant LSBGs, i.e., dwarf LSBGs are not sampled at all. Some of these works also use galaxies whose size or surface brightness are near the survey limits and are obliged to statistically correct their sample for missed galaxies by adopting simplifying assumptions, for example, that LSBGs have perfect exponential surface brightness profiles. The same assumption is done again in computing the volume correction (for field surveys).

Our own sample is a bit different from previous surveys: first of all, it is a volume-limited sample, since it is a cluster sample. Unlike previous cluster surveys, we do not impose a large minimal size (say 30″ as in Impey et al. 1988 for Virgo candidate galaxies), but we select the sample by absolute magnitude. Therefore, *dwarf* LSBGs are not discarded ab initio by adopting a large angular diameter for galaxies, provided that their flux brighter than the isophotal threshold is larger than the magnitude of completeness. Furthermore, previous LSBG searches compute the LSBG contribution to the LF *assuming* that all detected LSBGs belong to the studied cluster, while we compute the background and foreground contribution by using a control field. With respect to field LSBG searches, the advantages of the present determination are even larger: first of all, our sample is, as explained, a magnitude-complete sample, while field samples are often diameter-selected at a large angular diameter (for example, the survey of O'Neil, Bothun, & Cornell 1997, which has a similar depth, uses a 143 arcsec$^2$ minimal size, but at a 2–3 mag deeper isophote). Second, we choose to work only with the high S/N part of the catalog, thus completely skipping the problem of the correction for missed detections near the survey limits (minimal size and minimal brightness). Most importantly, the sample is volume-complete, and no volume correction/selection function should be computed, since the visibility of Coma galaxies does not depend on the redshift. It is true that besides Coma LSBGs there are other LSBGs in the Coma line of sight, but these are removed statistically from the sample. We remind the reader that the calculation of the volume correction/selection function for a diameter + brightness selected (field) sample is so difficult that many experienced astronomers, including Disney (1976), got it wrong when computing it (Disney 1999). This correction is quite large and thus uncertain. For example, the median incompleteness correction applied in the calculation of the LF of LSBGs by Sprayberry et al. (1997) is 5, which means that one detected object has been used to infer the presence of four other galaxies escaping detection or redshift determination with similar photometric parameters and in the same universe volume.

Sprayberry et al. (1997) find a steep LF for LSBGs, steeper than the LF usually found in samples claimed to be flux-limited. Their LF is computed in the $B$-band and concerns galaxies with $\mu_B(0) > 22$ mag arcsec$^{-2}$. Assuming that field LSBGs have an average $B-V = 0.5$ mag (e.g., de Blok et al. 1995), we can compute the $V$-band field LF for galaxies with $\mu_V(0) > 21.5$ mag arcsec$^{-2}$ from the $B$ LF. The latter cut in surface brightness is applied to the Coma galaxies in order to compare the two LFs. Figure 7 compares the result of this exercise. There is a remarkable agreement on the location and slope of the exponential decrease of the two LFs. However, the two LFs are arbitrarily shifted. The minor difference at $V < 17.5$ mag concerns 3.7 galaxies missed in the present LF that can be fully accounted for by stat-
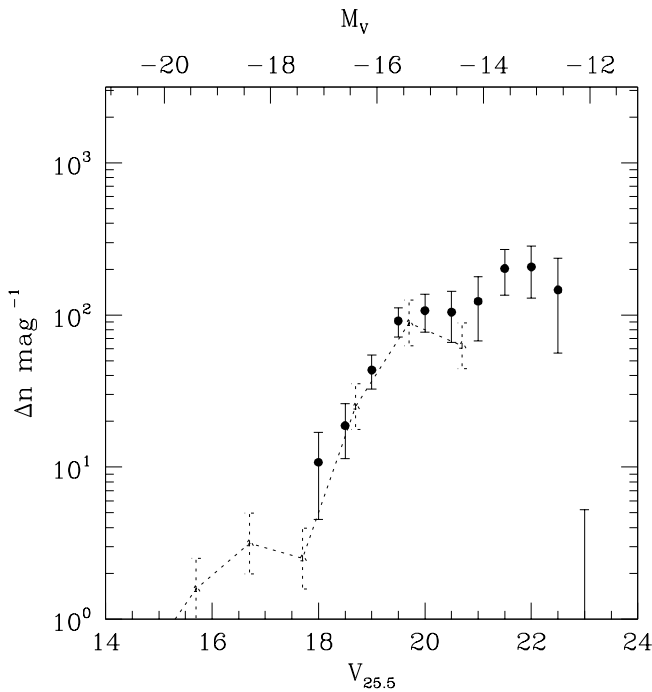
FIG. 7.—Comparison LF of field LSBGs (*dotted line and error bars*, taken from Sprayberry et al. 1997), transformed from the *B*-band (see text for details) to the LF of Coma LSBGs (*filled circles and solid error bars*). The amplitude of the field LF has been vertically shifted to match the much denser Coma Cluster.

istical fluctuations and by the difference in the passband used for selection. If the LSBGs LF is independent of the environment, this agreement confirms the correctness of the Sprayberry et al. (1997) calculation of the visibility function (actually, their survey has a selection function even more complex than those of diameter + brightness–limited surveys) and gives support to their claim that local field surveys overlook a numerous population of LSBGs.

## 6. CONCLUSION

Wide-field images of a nearby cluster, coupled with an exhaustive analysis of the sources of error, allow us to extend the LF to very faint magnitudes and to include the contribution of LSBGs. Most importantly, the data allow the determination of the bivariate LF, without missing LSBGs (down to a faint limiting central brightness) or losing compact galaxies, because of their resemblance to stars or to background galaxies. The present bivariate LF determination has a straightforward selection function allowing a precise measure of the frequency defining how galaxies occupy the available space in the central surface brightness versus magnitude plane in the Coma Cluster. Furthermore, the present determination does not need uncertain corrections for passing from the observed distribution to the actual galaxy distribution, simply because the sample is naturally volume-limited, or uncertain assumptions about the membership of faint galaxies, because the foreground and background have been statistically removed. LSBGs are by far the largest galaxy population; most of them are also quite faint, and this study suggests that if there is a minimal flux or central brightness, we have not yet reached it. On the other hand, compact galaxies are a minority population.

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