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LIGHT CURVES AND ELEMENTS OF AR DRACONIS

P. BROGLIA and P. CONCONI
Milano-Merate Astronomical Observatory, Italy

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B and *V* light curves of the eclipsing system AR Dra were obtained during the years 1976 and 1977. A small seasonal change in the light curves can be seen. Ten new epochs of minimum and an improved value for the period are derived. The seasonal light curves were analysed using a similar-ellipsoid model and assuming a Roche geometry.

Key words: eclipsing binaries – photometry

1. LIGHT CURVES AND PERIOD

AR Dra = BV 226 was discovered to be an eclipsing variable by Strohmeier and Geyer (Strohmeier 1959a) who gave a finding chart. A photographic light curve, fourteen epochs of minimum and the following light elements were afterwards reported by Strohmeier (1959b):

$$\mathrm{Min} = \mathrm{JD}\ 2426751.538 + 0\overset{\mathrm{d}}{.}675839\ n$$

The photoelectric observations studied in this article were obtained by means of the 102 cm reflector of the Merate Observatory during two observing seasons in 1976 and 1977. A Lallemand photomultiplier and *B*, *V* filters were used. The instrumental magnitudes are close to the corresponding in the *UBV* system. The photocurrent was fed into a Gardiner-type integrator with an integration time of 20 sec.

In figure 1 the comparison *c* and the check stars *a* and *b* are shown. The magnitudes differences between *c* and *a* gave a constant result, with a standard deviation for a single measurement of $\pm 0\overset{\mathrm{m}}{.}013$ in both the colours, whilst the star *b*, also referred to *c*, reckoning to the standard deviation of $\pm 0\overset{\mathrm{m}}{.}02$ and $0\overset{\mathrm{m}}{.}03$ respectively in *V* and *B*, appeared to be a little variable.

By means of a comparison to the *UBV* standards HD 105791 and HD 107582 made in three nights the magnitudes and the colours given in figure 1 were obtained. The estimated uncertainties were about $\pm 0\overset{\mathrm{m}}{.}01$. The values for the star *b* suffer of course from the limitation due to the suspected variability. Differential extinction between the variable and the comparison star was taken into account using mean extinction coefficients.

During 1976 a complete light curve based on about 1500 observations was obtained, but in the following year one thousand measurements allowed only a partial coverage of the light curve. There was however evidence of a little systematic change between the two seasonal light curves. The difference is noticeable mostly during the primary minimum as it is shown in the upper part of figure 2.

By means of a parabolic least squares fitting of the measurements around the deepest part of the eclipses ten epochs of minimum have been derived and are listed in table 1. The reported uncertainties are the mean errors calculated from *B* and *V* determinations. From the present data the following ephemeris and the residuals $O-C$ listed in table 1 were calculated by least squares:

$$\begin{array}{l} \mathrm{Min\ I} = \mathrm{helioc.\ J.D.}\ 2442868.91139 + 0\overset{\mathrm{d}}{.}67583575\ n \\ \qquad\qquad\qquad\qquad\qquad\quad \pm \qquad 3 \qquad\qquad\quad 10\ \mathrm{m.e.} \end{array}$$

Half a weight was given to the instants of secondary eclipse. The oldest photographic epochs, taking into account their precision, can be represented satisfactorily by means of these light elements, so the period seems to be constant.

For each of the two observing seasons normal points have been derived, listed in tables 2 and 3 and represented in figure 2; *n* is the number of single measurements pertaining to each normal. In the following the 1976 and 1977 normals are indicated respectively as V_1, B_1 and V_2, B_2 groups. The individual observations have been placed in the Variable Star Archives of the Royal Astronomical Society, file IAU (27). From the mean light curves it results moreover:

	1976		1977	
	V	$B-V$	V	$B-V$
Max	$11^{\mathrm{m}}.29$	$+0^{\mathrm{m}}.49$	$11^{\mathrm{m}}.31$	$+0^{\mathrm{m}}.49$
Min I	12.51	.65	12.47	.64
Min II	11.60	.46	11.63	.46

2. PHOTOMETRIC SOLUTIONS

Solutions of the light curves based on the Russell and on the Roche models have been attempted. It seems reasonable to assume that the changes in the light curves can be ascribed to the presence of circumstellar matter or to an intrinsic variability of one or both components. Therefore all the observations in each colour have been considered together in the solutions, imposing the same geometrical elements whilst the luminosities, during the two seasons, can be different.

The results obtained, assuming that the components of the system are two similar ellipsoids, are summarised in tables 4 and 5, where the usual notation is followed. The harmonic analysis of the normal points outside the eclipses, whose limits were adjusted after a preliminary solution, has been performed considering the terms up to the fifth order and disregarding the insignificant terms. The observations of the two seasons have been considered together and the calculated standard deviation for a normal, $0^{\mathrm{m}}.005$ in both the colours, agrees well with the estimated precision of the observations. After conventional rectifications of the normals were made with the constants given in table 4, least squares corrections to a set of preliminary elements has been calculated according to the method of Irwin (1947), by means of a PDP 11/10 computer. The α-functions and the corresponding derivatives with respect to k and p have been calculated following Jurkevich (1970). The elements i, r_g, r_s, L_g have been adjusted, with the luminosity L_g separately for each observing season. In table 5 the solutions calculated from the B and from the V light curves are reported and in figure 3 the residuals of the normals are represented against the phases. The primary minimum is shown to be a transit and the eclipses are deep, as $p_0 = -0.86$.

The solutions according to the Roche geometry were obtained by means of a programme developed by Wilson and Devinney (1973). In the first place the normals were corrected for the small asymmetry represented by the sinus terms of the Fourier analysis. No spectrographic observation of the system is known and in particular the ratio of masses q is lacking. Moreover when fitting the normals with theoretical light curves, calculated using the programme, it appeared that the fittings were not very sensitive to the value assumed for q. Because of the seasonal variation and the asymmetry we thought it suitable to estimate differently a value for q.

Taking into account the colour curve and the solutions given in table 5, the brighter and larger component appears to be an F5 star and to the smaller secondary star a spectral type near G8 can be assigned. Moreover, taking into account the radii derived in the Irwin solutions, according to the statistical relations (Kopal 1959) it appears that if the F-component is on the main sequence also the G star is there. According to this hypothesis a mass ratio near 0.75 can be estimated for the system (Kopal 1959). This value has been kept constant when finding a solution and moreover the same darkening coefficients used in the Russell model computations and reported in the table 4 have been used.

Initial elements were derived and the influence on the solutions of the different parameters was studied. It was shown that the bolometric albedo A_1 and the gravity coefficient g_2 have a negligible influence and therefore they have been kept constant. Then differential least squares correction procedure was used to obtain the optimal parameter set. The elements i, Ω_1, Ω_2, L_1, A_2, g_1 were optimized, considering simultaneously the normals of the two groups, V_1+V_2 and B_1+B_2, but adjusting separately the luminosities relative to the two seasons, as in the Russell computations. The programme was used in mode 0, with a grid of 19 × 19 elements. The solutions are given in table 6 and the residuals are reported in figure 3.

3. CONCLUDING REMARKS

The components of AR Dra are well inside the critical inner lobes corresponding to the assumed value $q=0.75$ and the system likely is a detached one. The elements calculated by means of the two methods agree well and give residuals consistent with the observational uncertainty. The seasonal changes of the light curves, when computing the elements according to the Russell model and to the assumptions mentioned above, give rise to a variation of the luminosities of the components. However in the Wilson and Devinney model in addition to L the parameters g_1 and A_2 also undergo temporal variations and were adjusted when performing the differential corrections.

Since the proximity effects in the system are small, judging by the Fourier coefficients, we can expect the two models substantially represent the system with the same accuracy and the corresponding solutions give in effect the same mean residuals. However a more consistent systematic trend can be noted in the distribution of the residuals (figure 3) when solving by means of the Russell method, particularly during the primary minimum.

The small variation in the light curves, of the order of a few hundreths of magnitude, give evidence of the existence of some sorts of activity in the system. The small sin ϑ terms in the harmonic analysis reveal that the maximum after the primary eclipse was brighter during 1976, but the lack of a complete coverage of the light curve during 1977 prevents seeing if the perturbation persisted. According to the solutions the changes of the minima during the interval 1976-1977 amount to little more than 10% of the light of the cooler component. The colour variation is so small that is is not possible to associate the fluctuation with the hotter or with the redder component. The presence of material around the components, suggested by sin ϑ terms is in contrast with the detached nature guessed for the system. An alternative explication to the activity in the binary can be an intrinsic variability of one star.

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P. Broglia
P. Conconi

Osservatorio Astronomico
22055 Merate (Como) Italy

Table 2 B normal points of AR Dra

- 1976 -

Phase	B	n	Phase	B	n	Phase	B	n	Phase	B	n	Phase	B	n
0.65	13.159	7	27.14	12.060	7	77.75	11.799	7	183.58	12.057	7	333.25	12.067	7
1.80	13.136	7	28.83	12.028	7	80.79	11.792	7	190.96	12.010	7	336.30	12.149	7
3.07	13.088	7	30.31	11.993	7	83.35	11.792	7	197.90	11.972	7	338.33	12.197	7
4.13	13.048	7	31.76	11.972	7	85.50	11.796	7	205.36	11.925	7	339.90	12.258	7
5.24	12.995	7	33.56	11.944	7	87.86	11.788	7	209.59	11.905	7	341.44	12.315	7
6.35	12.923	7	35.56	11.914	7	90.89	11.792	7	213.51	11.860	7	342.56	12.361	7
7.77	12.855	7	38.01	11.884	7	93.90	11.780	7	217.38	11.858	7	343.88	12.414	7
8.67	12.779	7	41.15	11.867	7	97.58	11.790	7	221.74	11.850	7	345.19	12.472	7
9.65	12.730	7	44.59	11.854	7	102.17	11.787	7	226.91	11.841	7	346.37	12.534	7
10.76	12.664	7	47.52	11.847	7	108.15	11.789	7	231.86	11.847	7	347.38	12.575	7
12.24	12.593	7	50.27	11.838	7	114.32	11.799	7	237.30	11.834	7	348.69	12.646	7
13.65	12.527	7	54.29	11.832	7	120.87	11.814	7	248.16	11.810	7	349.85	12.708	7
15.06	12.468	7	56.89	11.825	7	126.92	11.818	7	258.60	11.807	7	350.84	12.768	7
16.15	12.413	7	58.94	11.821	7	133.64	11.827	7	264.28	11.807	7	352.10	12.830	7
17.46	12.359	7	61.25	11.823	7	139.51	11.839	7	271.60	11.796	7	353.11	12.896	7
18.34	12.307	7	63.91	11.816	7	147.78	11.872	7	281.37	11.815	7	354.24	12.948	7
20.18	12.260	7	66.68	11.813	7	152.38	11.906	7	292.64	11.822	7	355.38	13.023	7
21.77	12.204	7	68.62	11.810	7	155.28	11.914	7	307.85	11.839	7	356.74	13.080	7
23.12	12.169	7	70.89	11.802	7	159.04	11.949	7	318.52	11.878	7	357.78	13.120	7
24.48	12.128	7	73.48	11.803	7	165.29	11.963	7	324.98	11.916	7	358.87	13.150	7
25.90	12.088	7	75.57	11.815	7	174.64	12.029	7	329.35	11.979	7	359.82	13.166	5

- 1977 -

Phase	B	n	Phase	B	n	Phase	B	n	Phase	B	n	Phase	B	n
1.14	13.099	7	36.10	11.892	7	157.08	11.920	7	208.72	11.906	7	310.42	11.840	7
3.33	13.054	7	38.15	11.864	7	160.87	11.944	7	216.32	11.870	7	316.70	11.850	7
5.59	12.937	7	40.52	11.852	7	166.57	11.980	7	225.64	11.842	7	320.91	11.870	7
8.29	12.797	7	42.78	11.836	7	172.98	12.023	7	236.62	11.820	7	325.64	11.906	7
10.34	12.682	7	45.19	11.835	7	176.76	12.059	7	243.74	11.823	7	330.49	11.987	7
12.66	12.563	7	48.85	11.842	7	180.70	12.061	7	251.67	11.806	7	334.86	12.077	7
15.64	12.417	7	53.75	11.819	7	185.07	12.053	7	257.54	11.807	7	339.56	12.233	7
18.02	12.310	7	57.75	11.820	5	188.62	12.029	7	264.46	11.799	7	342.43	12.335	7
20.42	12.224	7	60.87	11.819	5	191.86	12.013	7	271.42	11.792	7	345.07	12.451	7
22.62	12.154	7	135.17	11.834	7	194.19	11.994	7	276.75	11.788	7	348.12	12.608	7
24.68	12.097	7	139.95	11.840	7	196.96	11.976	7	281.86	11.790	7	351.04	12.747	7
26.74	12.044	7	144.09	11.855	7	199.34	11.965	7	287.52	11.805	7	353.53	12.889	7
29.55	11.986	7	149.31	11.873	7	202.00	11.942	7	294.83	11.809	7	356.46	13.034	7
31.68	11.940	7	153.37	11.891	7	204.73	11.929	7	302.18	11.824	7	359.02	13.103	7
33.91	11.904	7												

Table 3 V normal points of AR Dra

- 1976 -

Phase	V	n	Phase	V	n	Phase	V	n	Phase	V	n	Phase	V	n
0.48	12.507	7	24.12	11.621	7	76.61	11.291	7	191.91	11.537	7	336.12	11.636	7
1.40	12.490	7	25.52	11.581	7	79.16	11.294	7	198.00	11.491	7	337.46	11.661	7
2.36	12.478	7	26.73	11.551	7	81.41	11.293	7	201.79	11.466	7	338.40	11.683	7
3.16	12.448	7	27.98	11.537	7	83.92	11.292	7	206.61	11.424	7	339.46	11.726	7
3.98	12.420	7	29.58	11.502	7	86.15	11.291	7	212.41	11.388	7	340.68	11.763	7
4.82	12.382	7	30.94	11.478	7	88.76	11.290	7	218.12	11.357	7	342.01	11.803	7
5.66	12.339	7	32.43	11.451	7	91.72	11.290	7	224.40	11.340	7	343.35	11.852	7
6.43	12.306	7	34.08	11.427	7	94.78	11.287	7	232.00	11.334	7	344.59	11.898	7
7.44	12.256	7	35.80	11.406	7	98.69	11.290	7	238.80	11.326	7	345.83	11.946	7
8.26	12.217	7	38.19	11.384	7	102.54	11.287	7	246.70	11.299	7	346.88	11.995	7
9.10	12.182	7	40.84	11.366	7	109.83	11.296	7	256.76	11.309	7	348.01	12.044	7
10.09	12.120	7	44.22	11.358	7	116.36	11.305	7	262.21	11.309	7	349.28	12.106	7
10.89	12.099	7	47.17	11.346	7	123.54	11.316	7	274.79	11.309	7	350.43	12.162	7
11.87	12.053	7	49.84	11.344	7	130.71	11.323	7	291.34	11.328	7	351.54	12.207	7
12.86	12.007	7	53.65	11.334	7	136.98	11.338	7	307.44	11.344	7	352.53	12.245	7
13.94	11.959	7	56.89	11.325	7	143.64	11.353	7	314.21	11.347	7	353.28	12.286	7
15.29	11.907	7	59.36	11.321	7	151.12	11.408	7	318.16	11.369	7	354.21	12.333	7
16.33	11.868	7	62.41	11.321	7	153.97	11.409	7	322.47	11.402	7	355.09	12.382	7
17.72	11.818	7	65.44	11.318	7	157.05	11.446	7	325.95	11.443	7	355.92	12.424	7
19.03	11.777	7	67.98	11.312	7	162.19	11.485	7	329.20	11.483	7	356.86	12.458	7
20.31	11.733	7	70.10	11.308	7	169.61	11.539	7	331.09	11.509	7	357.65	12.473	7
21.67	11.688	7	72.35	11.297	7	177.66	11.598	7	332.81	11.552	7	358.61	12.495	7
22.93	11.651	7	74.72	11.302	7	185.34	11.594	7	334.42	11.591	7	359.55	12.509	7

- 1977 -

Phase	V	n	Phase	V	n	Phase	V	n	Phase	V	n	Phase	V	n
0.81	12.466	8	35.76	11.387	7	166.03	11.533	7	208.70	11.416	7	317.03	11.355	7
3.40	12.405	7	37.97	11.365	7	172.96	11.601	7	216.39	11.375	7	321.73	11.373	7
5.59	12.314	7	40.24	11.367	7	177.04	11.630	7	226.75	11.339	7	326.08	11.415	7
8.15	12.199	7	42.48	11.344	7	179.34	11.632	7	237.56	11.324	7	331.01	11.488	7
10.21	12.100	7	44.93	11.335	7	182.53	11.612	7	245.09	11.331	7	335.74	11.591	7
12.44	11.998	7	48.62	11.330	7	184.58	11.601	7	252.20	11.318	7	340.33	11.719	7
15.19	11.882	7	54.14	11.318	7	186.50	11.590	7	257.48	11.316	7	342.85	11.807	7
17.69	11.791	7	59.49	11.314	8	188.36	11.573	7	263.47	11.302	7	345.19	11.904	7
20.06	11.705	7	134.39	11.327	7	189.99	11.563	7	270.31	11.311	7	348.38	12.039	7
22.45	11.638	7	139.58	11.345	7	192.12	11.546	7	275.85	11.304	7	351.54	12.185	7
24.49	11.588	7	143.68	11.369	7	194.37	11.521	7	280.89	11.303	7	353.80	12.296	7
26.36	11.550	7	147.97	11.398	7	197.14	11.496	7	286.50	11.320	7	356.29	12.393	7
28.88	11.492	7	152.39	11.410	7	199.47	11.482	7	294.66	11.328	7	358.55	12.457	7
31.20	11.447	7	156.10	11.431	7	202.35	11.451	7	302.01	11.327	7			
33.66	11.411	7	160.01	11.467	7	204.83	11.434	7	310.26	11.347	7			

Table 1 Times of minima of AR Dra

Helioc. J.D.	m.e.	n	O-C
242869.58672	$\pm 0^{d}.00002$	1	$-0^{d}.00051$
2901.35153		48	+.00002
2903.37925	.00005	51	+.00023
2904.3946	.0005	52.5	+.0018
2905.40648	.00018	54	-.00004
2930.41244	.00001	91	-.00001
2932.43992	.00012	94	-.00003
3247.37941	.00006	560	-.00001
3249.40691	.00007	563	-.00002
3250.4206	.0014	564.5	-.0001

Table 4 Fourier coefficients and rectification constants

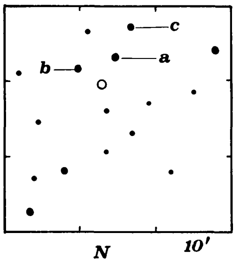
Group	A_0	A_1	A_2	B_1	B_2	C_0	z
V_1+V_2	.9391	-.0084	-.0397	.0072		.0246	.076
	± 14	16	20	8			
B_1+B_2	.9261	-.0047	-.0431	.0042	-.0038	.0345	.079
	15	17	21	8	10		

Table 5 Solutions for AR Dra calculated according to the Russell model

Group	L_g	i_r	r_g	r_s	x_g	x_s	m.e.
V_1	.815						$0^{m}.006$
	± 3						
		$84^{\circ}.8$	.377	.325	.60	.70	
		3	2	1			
V_2	.797						.008
	3						
B_1	.870						.007
	3						
		$84^{\circ}.0$	.380	.325	.80	.80	
		2	2	1			
B_2	.855						.007
	3						

Table 6 Solutions for AR Dra calculated according to the Roche model

Group	L_1	g_1	A_2	i	Ω_1	Ω_2	r_{1pl}	r_{1pt}	r_{1bk}	r_{1sd}	r_{2pl}	r_{2pt}	r_{2bk}	r_{2sd}	σ
V_1	.819	.4	.64												$0^{m}.006$
	± 6	1	7												
				$84^{\circ}.9$	3.645	3.554	.341	.388	.370	.354	.303	.352	.334	.314	
				2	20	12	2	4	3	3	1	3	2	2	
V_2	.789	.0	.37												.007
	5		7												
B_1	.871	.22	.65												.007
	5	7	6												
				$84^{\circ}.7$	3.623	3.569	.343	.392	.374	.357	.302	.349	.331	.312	
				2	13	9	2	3	2	2	1	2	2	1	
B_2	.845	.0	.48												.007
	3		7												



	V	B-V
a	10.68	+ 0.67
b	10.30	+ 1.56
c	10.84	+0.58

Figure 1 Comparison and check stars for AR Dra.

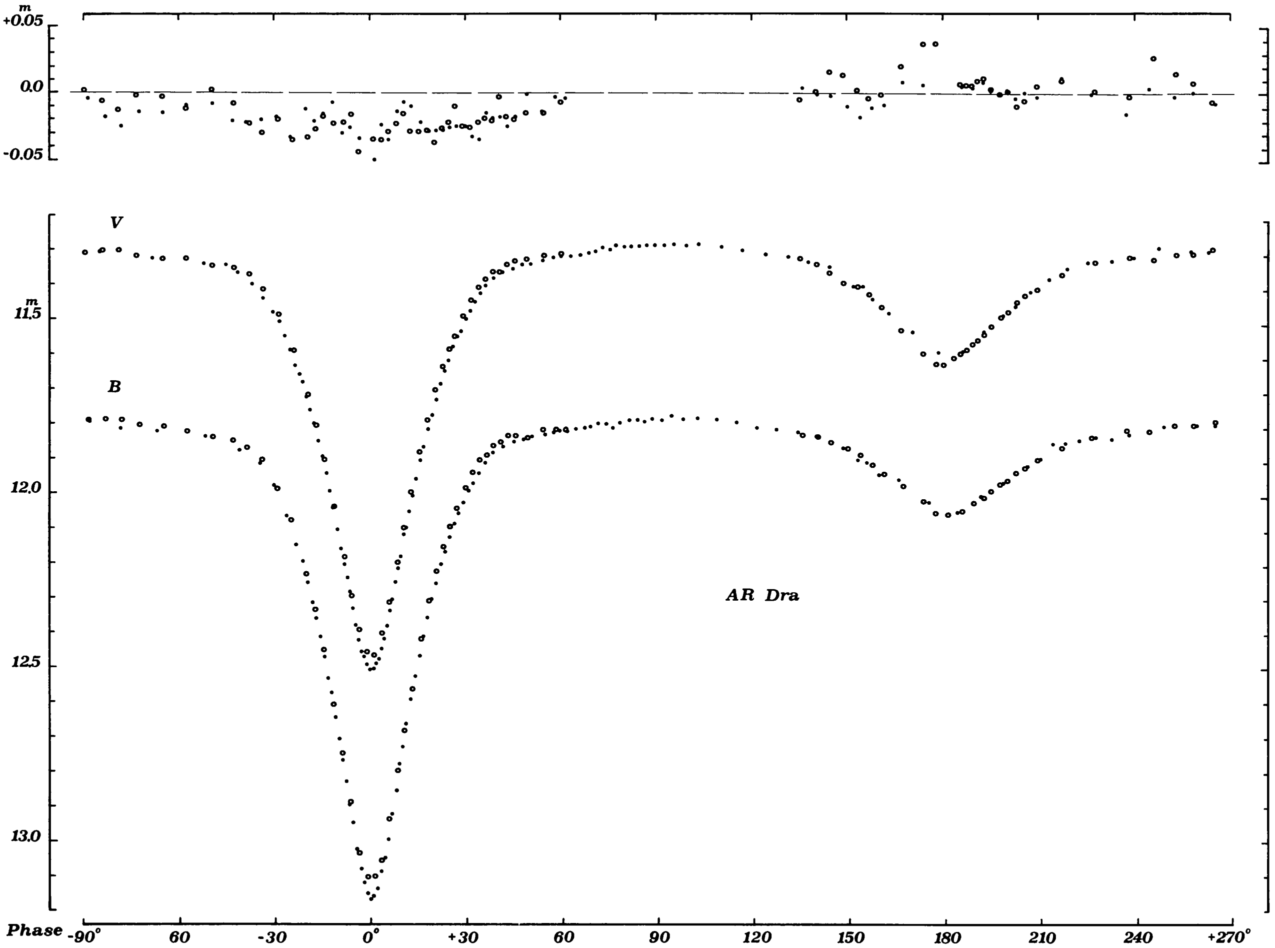


Figure 2 Seasonal light curves for AR Dra. The dots and the circles represent respectively the normals obtained during 1976 and 1977. In the upper part the seasonal differences between the light curves *B* (dots) and *V* (circles) are shown.

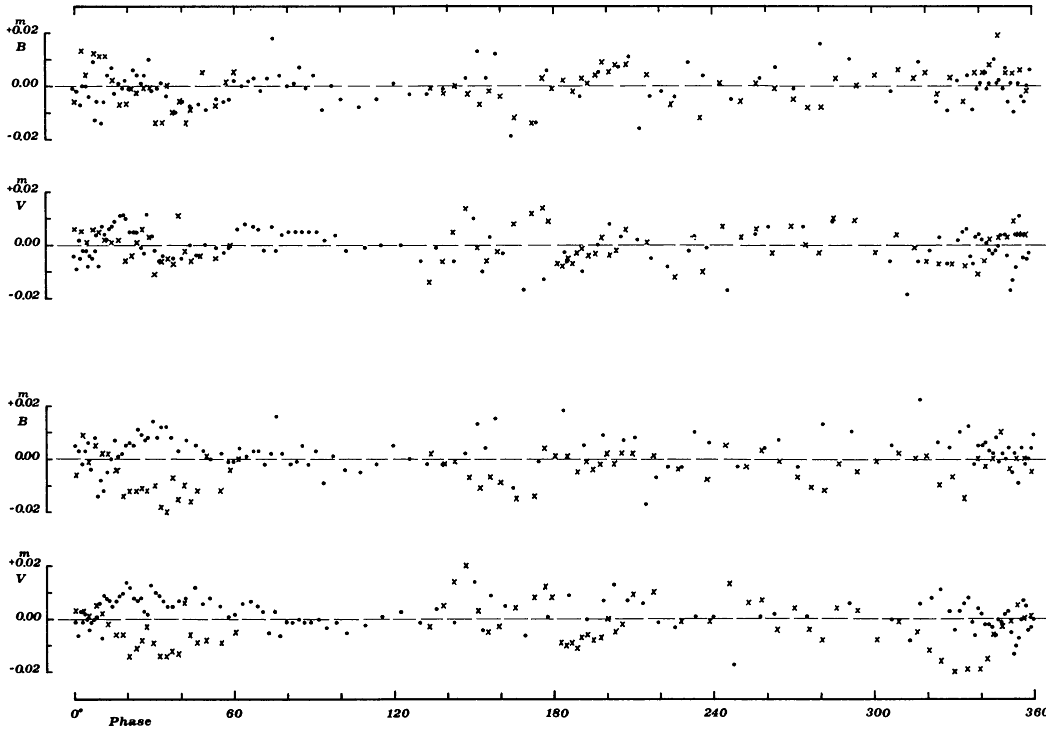


Figure 3 Plots, against the phases, of the residuals calculated according to the Russell and Merrill (1952) method (lower part), and by means of the Wilson and Devinney (1971) programme. Dots refer to the 1976 observations, crosses to the 1977 ones.