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Study of the variability of the δ Scuti stars

VIII. HR 4684 and the resonance mechanism in dwarf pulsators

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Summary. The light curve analysis of HR 4684 provides an observational indication of the presence in δ Scuti stars of the parametric resonance mechanism suggested by Dziembowski. Probably, this process plays a general rôle in the pulsation of dwarf stars, by exciting low order modes which, in a linear approach, should be stable, and limiting the pulsational amplitudes. Physical features and pulsational modalities, both in HR 4684 and in another δ Scuti star (KW 207), agree with the assumption of the presence of parametric resonance phenomena in normal δ Scuti stars.

Key words: δ Scuti stars – oscillations of stars

1. Introduction

The mechanism which limits the amplitude is a crucial problem for the understanding of the pulsational behaviour of dwarf variables. The physical explanation of the phenomenon is not known, and, moreover, there are complicated connections among the observed amplitude, rotation and metallicity (Breger, 1972; Kurtz, 1976; Baglin et al., 1980; Breger, 1982a). Nevertheless, the sharing of pulsational energy among several modes is probably a very important factor limiting the amplitude observed (Dziembowski, 1980). Dziembowski (1982) has shown that this distribution of energy may be due to nonlinear mode coupling. In particular, he discriminates between a direct resonance mechanism and a parametric one. In the first case, one or two unstable modes feed a resonant mode with a higher frequency. In the parametric resonance case, a high frequency unstable mode partially decays in one or two damped lower frequency modes.

A probable observational example of parametric resonance is given by KW 207 (HD 73576 – Bossi et al., 1982). In fact, the analysis of the light curve of this δ Scuti star has given the possible periods $P1 = 0\overset{\mathrm{d}}{.}0583$, $P2 = 0\overset{\mathrm{d}}{.}0642$ and $P3 = 0\overset{\mathrm{d}}{.}1175$, with $P3 \simeq 2 \cdot P1$. The $Q1$ value ($\simeq 0.019$) suggests a relatively high overtone or a high order nonradial p-mode, $Q2$ ($\simeq 0.021$) a lower overtone or nonradial p-mode and $Q3$ ($\simeq 0.038$) the fundamental radial mode, a f-mode or a g_l ($l \simeq 3$)-mode. Since the component with the shortest period ($P1$) has the highest amplitude, we could interpret the mode with the period $P3$ as a damped mode which is excited by resonance.

In the present paper we discuss a second possible example of resonance mechanism, HR 4684, a δ Scuti star observed at the Brera-Merate Observatory in the framework of our research program on the variability of this kind of objects. In spite of the unavoidable uncertainties in the light curve analysis of δ Scuti stars, this new observational indication leads us to think that the mechanism suggested by Dziembowski really works in δ Scuti stars. A preliminary discussion on the photometric variability of HR 4684 has been reported by Antonello et al. (1984).

2. The light curve of HR 4684

The member of the Coma cluster HR 4684 is a δ Scuti star which lies near the blue edge of the instability strip. Elliot (1974) observed this star for few nights in 1970, and derived the tentative period $P = 0\overset{\mathrm{d}}{.}0551$. This object was observed by us in 1976, and $177\,\Delta m_V$ normal points distributed over 12 nights were obtained (Guerrero and Mantegazza, 1976). The comparison and the check stars were HD 107326 and HD 107655, respectively. For the instrumental equipment and reduction procedure, see Bossi et al. (1977). Our measurements are reported in Table 1 and the light curve is shown in Fig. 1.

Two methods of analysis were adopted: the least-squares power spectrum proposed by Vanicek (1971) and the nonlinear least-squares period determination routine *PERDET* (Breger, 1982b) in connection with the Fourier transform for unequally spaced data (Deeming, 1975). The results obtained by means of both methods are very similar: we find three sinusoidal components of comparable amplitudes. However, there is an uncertainty about the exact values of their frequencies, because of the data spacing, which, as one can see from the spectral window (Fig. 2), produces two strong aliases at ±1 cycle/day. The problem is entangled by the presence of some power at low frequencies, mainly due to different mean light levels for different nights. Hence we have several sets of frequencies, which are equivalent from the least-squares point of view:

10.25, 15.02 and 25.23 c/d;

9.25, 15.02 and 24.27 c/d;

10.25, 16.02 and 25.23 c/d;

9.25, 16.02 and 25.23 c/d;

9.31, 14.98 and 24.27 c/d.

We do not confirm the period found by Elliot. Taking into account the structure of the frequency patterns we give the first set the preference. Figure 3 (top) shows the power spectrum of the observational data, where the mean values of all the nights were

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Table 1

Hel. J. D. 2400000+	ΔV	σ	Hel. J. D. 2400000+	ΔV	σ	Hel. J. D. 2400000+	ΔV	σ
42838.430	-0.287	.006	.493	-0.287	.003	.385	-0.299	.001
.435	-0.289	.005	.502	-0.286	.001	.398	-0.276	.003
.441	-0.295	.006	.511	-0.294	.002	.403	-0.279	.002
.446	-0.294	.004	.519	-0.299	.001	.408	-0.275	.004
.452	-0.289	.003	.529	-0.288	.002	.420	-0.284	.001
.457	-0.290	.007	.544	-0.292	.002	.426	-0.290	.002
.467	-0.290	.003	.552	-0.298	.005	.431	-0.290	.002
.476	-0.296	.002	42866.505	-0.284	.002	.443	-0.290	.002
.483	-0.301	.003	.513	-0.285	.003	.449	-0.285	.005
.489	-0.298	.004	.522	-0.277	.003	.456	-0.289	.004
.496	-0.288	.002	.530	-0.273	.002	.470	-0.291	.003
.502	-0.279	.003	.538	-0.290	.003	.475	-0.286	.002
.508	-0.281	.002	.545	-0.289	.002	.480	-0.287	.002
.514	-0.292	.003	.553	-0.282	.001	.492	-0.290	.002
.520	-0.297	.001	.561	-0.276	.001	.497	-0.285	.004
.526	-0.312	.003	.570	-0.283	.002	42915.340	-0.273	.004
.531	-0.301	.005	.578	-0.295	.002	.347	-0.274	.004
.536	-0.300	.006	.586	-0.301	.002	.352	-0.281	.002
.540	-0.288	.003	.594	-0.294	.002	.358	-0.284	.004
.545	-0.293	.002	.610	-0.280	.002	.372	-0.299	.003
.550	-0.289	.005	.618	-0.292	.002	.378	-0.296	.002
.557	-0.288	.003	.625	-0.291	.002	.384	-0.286	.002
.561	-0.296	.006	42912.438	-0.293	.002	.397	-0.270	.001
.566	-0.291	.002	.443	-0.296	.003	.403	-0.270	.001
.572	-0.290	.003	.449	-0.288	.003	.408	-0.273	.001
.577	-0.291	.003	.454	-0.287	.003	.420	-0.276	.001
.583	-0.298	.001	.459	-0.283	.003	.425	-0.275	.002
.589	-0.295	.005	.466	-0.278	.002	.431	-0.275	.001
.595	-0.303	.003	.471	-0.280	.002	.435	-0.275	.001
.602	-0.298	.005	.477	-0.280	.002	.450	-0.309	.007
42862.405	-0.285	.003	.482	-0.289	.003	.460	-0.297	.003
.410	-0.284	.003	.490	-0.282	.002	.466	-0.288	.001
.425	-0.291	.003	.496	-0.282	.004	.473	-0.289	.003
.430	-0.291	.001	.501	-0.279	.004	.478	-0.288	.006
.442	-0.284	.001	.507	-0.293	.004	42916.334	-0.271	.003
.446	-0.288	.003	.512	-0.283	.002	.339	-0.273	.002
.460	-0.290	.004	.517	-0.281	.002	.344	-0.269	.002
.465	-0.290	.003	.522	-0.281	.003	.352	-0.263	.002
.476	-0.278	.004	.528	-0.285	.004	.359	-0.276	.002
.480	-0.274	.003	.533	-0.280	.004	.371	-0.288	.003
.499	-0.298	.005	42913.334	-0.279	.002	.376	-0.271	.002
.516	-0.287	.001	.341	-0.279	.003	.382	-0.259	.002
.520	-0.287	.005	.355	-0.284	.002	.388	-0.256	.001
.534	-0.284	.004	.362	-0.286	.001	.401	-0.265	.001
.538	-0.284	.003	.376	-0.282	.002	.407	-0.270	.001
.549	-0.291	.003	.382	-0.288	.003	.413	-0.268	.001
.553	-0.293	.005	.387	-0.292	.002	.419	-0.268	.002
.564	-0.308	.005	.399	-0.287	.002	.425	-0.272	.004
.567	-0.293	.003	.405	-0.289	.003	.438	-0.292	.003
.578	-0.295	.003	.410	-0.288	.003	.444	-0.293	.003
.582	-0.294	.003	.422	-0.292	.006	.449	-0.283	.002
.596	-0.291	.005	42914.329	-0.282	.005	.454	-0.267	.004
42863.445	-0.281	.002	.335	-0.278	.002	.459	-0.279	.006
.457	-0.289	.002	.340	-0.280	.001	.464	-0.257	.007
.474	-0.293	.001	.353	-0.281	.002	42917.330	-0.274	.001
42865.449	-0.280	.003	.358	-0.280	.001	.335	-0.273	.002
.458	-0.282	.002	.363	-0.284	.004	.348	-0.278	.001
.468	-0.291	.001	.375	-0.299	.002	.354	-0.279	.002
.482	-0.298	.003	.380	-0.299	.001	.360	-0.278	.002

normalized. The spectrum of a synthesized light curve is represented in the same figure (bottom); this curve was obtained using the 10.25 c/d + 15.02 c/d + 25.23 c/d solution without addition of noise and sampled as the observations (the curve is also shown in Fig. 1).

Because of the uncertainties in the period determinations, we cannot obtain reliable mode identifications. Nevertheless, some indications are given by the Q values. From the photometric indices of this star (Hauck and Mermilliod, 1980), through Crawford's calibration (1979), we derive $(b-y)_0=0.098$, $(c_1)_0=0.946$, and $M_V=1.71$. Using Breger's calibration (1977) and a bolometric correction (Hayes, 1978), we obtain $T_e=8350$ K, $\log g=4.05$, and $M_{\rm bol}=1.68$. Hence we have $Q1\simeq0.045$, $Q2\simeq0.031$, and $Q3\simeq0.018$. The high value of $Q1$ suggests a non radial g_1^3-mode (Fitch, 1981), $Q2$ the radial fundamental mode and $Q3$ a relatively high overtone ($3H$ or a nonradial p_3^1, p_3^2-mode).

In any case, it is very interesting to note that most of our solutions verify the relation $f3=f1+f2$, where $f1$, $f2$, and $f3$ are the three frequencies. This fact is strengthened if we consider only the decimal place of all the proposed solutions: in this case, the above sum is always verified within the uncertainties. It is not correct to estimate the accuracy of the frequency determinations by considering the whole time basis, because 54% of the observations were made during six consecutive nights; taking this into account, we derive an uncertainty in the frequencies of about 0.04 c/d.

The semi-amplitudes of our three components are of about 0.004–0.005 mag. It seems that $f3$ has the highest amplitude, hence we could interpret this component as an unstable mode which decays in two lower order damped modes. In particular, one of these modes may be a g-mode. The fit of the computed solution to

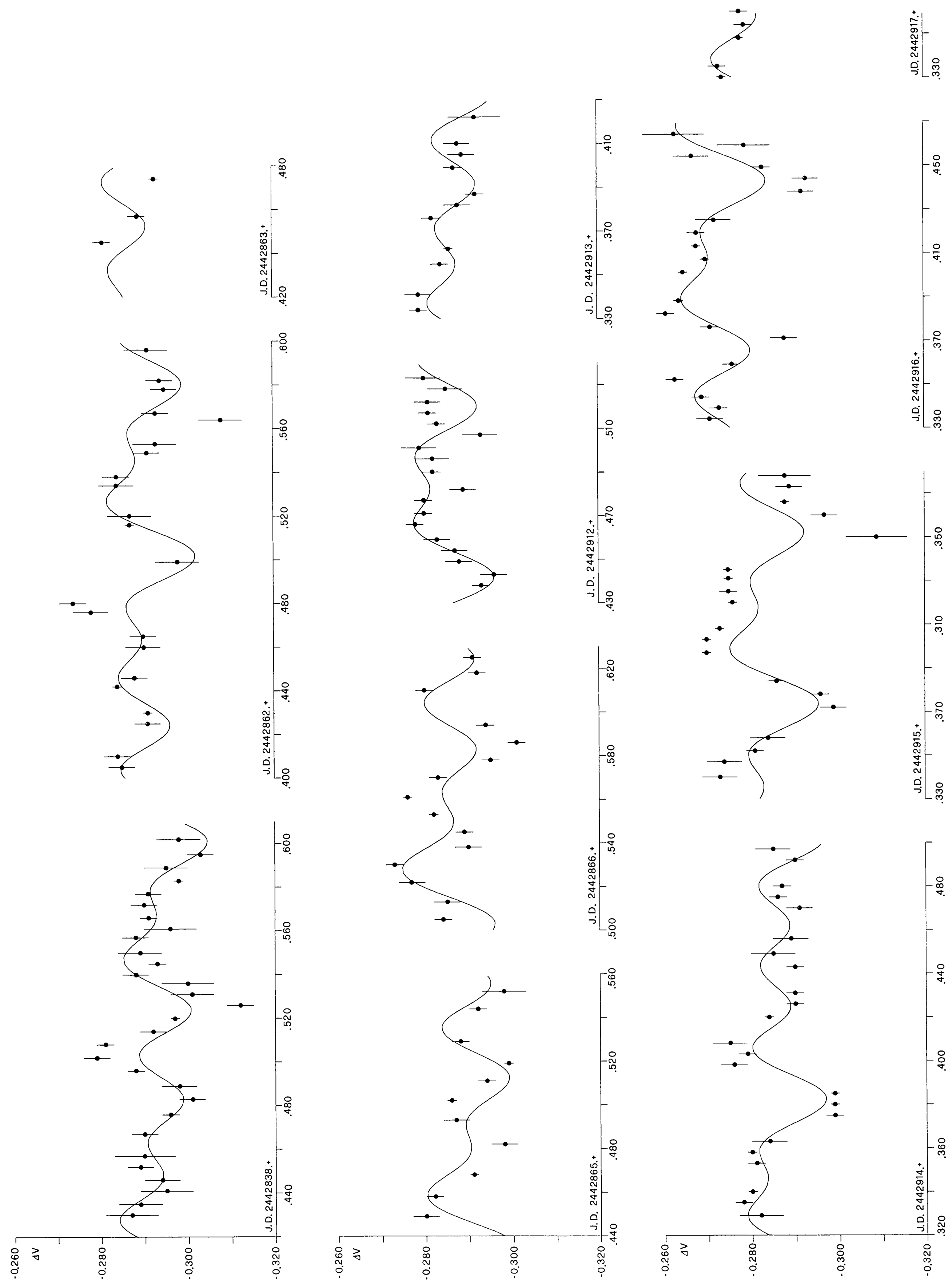

Fig. 1. The ΔV light curve of HR 4684. Bars represent the standard errors of the normal points. The solid line is the synthesized light curve obtained for the 10.25 c/d + 15.02 c/d + 25.23 c/d solution

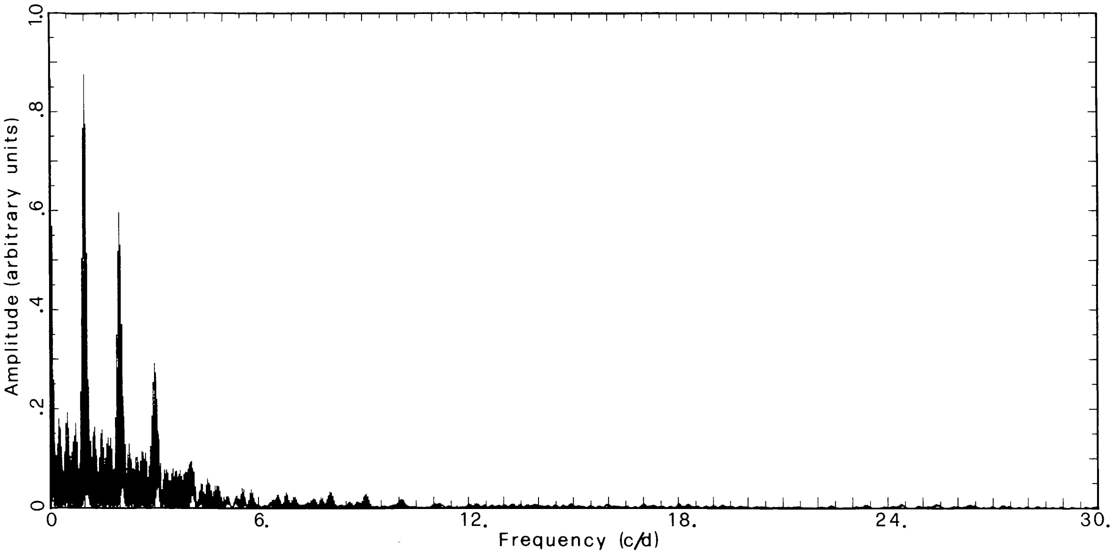

Fig. 2. The spectral window corresponding to our data sampling

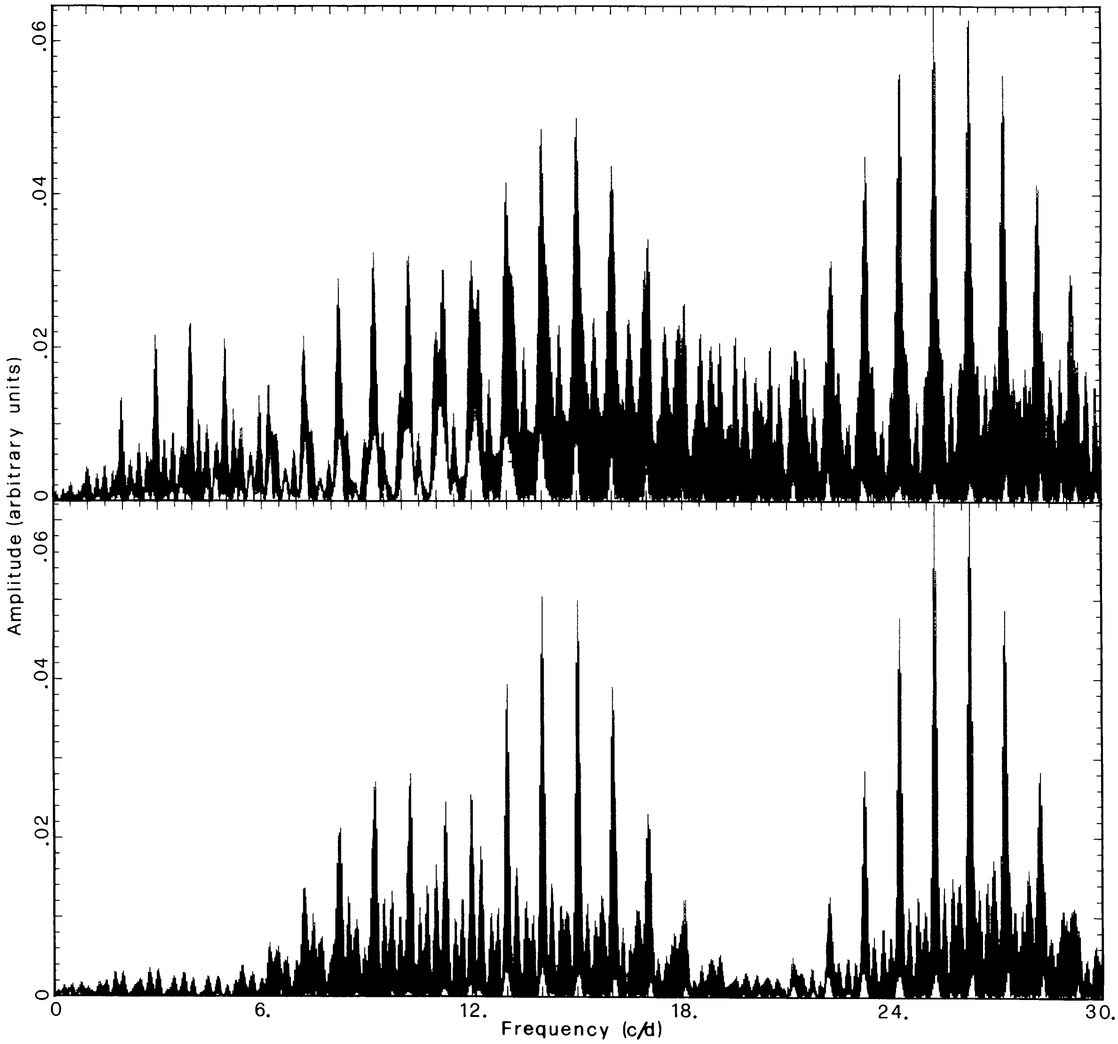

Fig. 3. The power spectrum of the observational data (top); the mean values of all the nights are normalized. The power spectrum of the synthesized light curve of Fig. 1, sampled as the observed one (bottom)

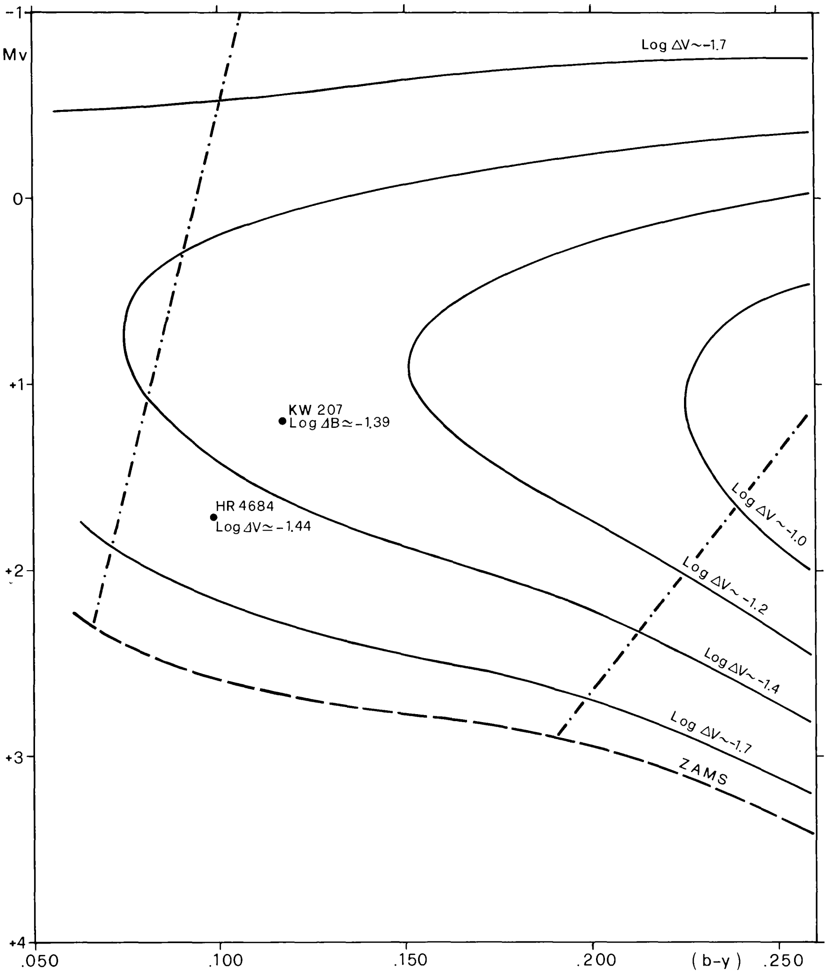

Fig. 4. The representative points of HR 4684 and KW 207 in a colour-magnitude diagram. The maximum pulsational amplitude of KW 207 is available only in *B* colour; nevertheless, *B* and *V* amplitudes are generally comparable. The curves indicate approximately the position of stars with the same amplitude ΔV

the observations, even if it reproduces their trend, is not satisfactory. We think that this may be due partly to a possible variation of the amplitudes of the components.

3. The possible role of resonances

As a result of a detailed linear survey of stellar models in the framework of a study on radial pulsations, Stellingwerf (1979) concludes that, generally, the high overtones are the most unstable ones in δ Scuti stars. In particular the fourth and fifth modes exhibit both blue and red edges consistent with the observed instability strip. If the growth rates of low *l*-order nonradial modes are similar to those of radial ones (Dziembowski, 1977; Dziembowski, 1980; see also Fitch, 1981), we may conclude, more generally, that the radial and/or nonradial high overtones may be really excited in δ Scuti stars. Indeed this is verified for the modes interpreted by us as linearly driven both in HR 4684 and in KW 207.

In this case, the assumption of a possible rôle of a resonance mechanism such as the parametric one may become very attractive. Indeed, the pulsational amplitudes are small because of the sharing of the energy among several modes, and, furthermore, the nonlinear coupling may be such an efficient factor in removing energy from the linearly unstable modes, that the opacity driving mechanism is not saturated (Dziembowski, 1980). Clearly, this fact prevents the amplitude to increase to the full regime.

A preliminary indication of the interest of the mechanism for δ Scuti stars is provided by our two objects. In fact, physical features and pulsational modalities agree very well, both for HR 4684 and KW 207, with the relation found by Antonello (1984; see also Antonello et al., 1981). We show in Fig. 4 the representative points of HR 4684 and KW 207, with the indication of pulsational amplitudes, in a color-magnitude diagram. The ΔV amplitude curves derived by means of the relation:

$$\log \Delta V = 1.90 \delta c_1 M_V + 0.62 \log P - 1.00$$

are also represented. ΔV is the maximum observed amplitude of V light curve, δc_1 is a photometric index of luminosity (see, e.g., Crawford, 1975), M_V is the absolute magnitude and P is the period. The basis of Antonello's work is a statistical analysis of 44 normal, well observed δ Scuti stars with low amplitude. Hence, if the presence of a parametric resonance was a special case, one should expect amplitudes not consistent with the general rule.

Undoubtedly, new "aimed" observations, combined with a better theoretical understanding of the nonlinear phenomena, will tell us if we are, or not, on the right track.

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