Extending the Butcher–Oemler effect up to $z \sim 0.7$

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Accepted 2003 December 15. Received 2003 December 12; in original form 2003 September 11

ABSTRACT

We have observed three clusters at $z \sim 0.7$, of richness comparable to the low-redshift sample of Butcher & Oemler (BO), and have determined their fraction of blue galaxies. When adopting the standard error definition, two clusters have a low blue fraction for their redshifts, whereas the fraction of the third one is compatible with the expected value. A detailed analysis of previous BO–like studies that adopted different definitions of the blue fraction shows that the modified definitions are affected by contaminating signals: colour segregation in clusters affects blue fractions derived in fixed metric apertures, differential evolution of early and late type spirals potentially affects blue fractions derived with a non-standard choice of the colour cut, and the younger age of the universe at high redshift affects blue fractions computed with a colour cut taken relatively to a fixed non-evolving colour. Adopting these definitions, we find largely varying blue fractions. This thorough analysis of the drawbacks of the different possible definitions of the blue fraction should allow future studies to perform measures in the same scale. Finally, if one adopts a more refined error analysis to deal with BO and our data, a constant blue fraction with redshift cannot be excluded, showing that the BO effect is still far from being detected beyond doubt.

Key words: galaxies: clusters: general – galaxies: clusters: individual: J0048316–294206.6 – galaxies: clusters: individual: J2245132–395409.9 – galaxies: clusters: individual: J2249321–395804.6 – galaxies: evolution.

1 INTRODUCTION

Butcher & Oemler (1978, 1984, BO hereafter) provided the first dramatically clear evidence that galaxy populations differ at high and low redshifts: clusters at high redshift contain a larger fraction of blue galaxies than their nearby counterparts. Dressler et al. (1994) have shown, by using images from the refurbished *Hubble Space Telescope*, that the blue galaxies responsible for the BO effect in the particular case of cluster cl 0939+4713 at z = 0.41 are late-type spiral and irregular galaxies. Rakos & Schombert (1995) found that 80 per cent of the galaxies in clusters at z = 0.9 are blue, in clear contrast to 20 per cent at z = 0.4.

The abrupt variation in cluster colour content observed by Rakos & Schombert (1995) poses the problem of finding a highly efficient mechanism that can account for these galaxy transformations on such short time-scales. In fact, the authors comment on the difficulty to imagine a scenario where over 80 per cent of the cluster population is destroyed or faded, especially because no remaining evidence (some sort of counterparts) seems to be detected in nearby clusters,

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into another kind of galaxy type. While such a population cannot possibly transform into early-

and prefer a scenario where the high-z blue galaxies have evolved

type galaxies which are old, both locally (Bower, Lucey & Ellis 1992; Andreon 2003) and up to z = 1 (Andreon, Davoust & Heim 1997; Ellis et al. 1997; Kodama et al. 1998; Stanford, Eisenhardt & Dickinson 1998; but see van Dokkum & Franx 2001 for a different opinion), S0 galaxies could provide a destiny (but see Ellis et al. 1997; Andreon 1998b; Jones, Smail & Couch 2000 for a different opinion). The relative fractions of spirals and S0s observed in clusters at different redshifts (Dressler et al. 1997) seem to support such morphological transformations (but see Andreon 1998b; Lubin et al. 1998 for a different opinion).

However, Allington-Smith et al. (1993) argue that galaxies in groups do not evolve (except passively), at least over the redshift interval 0 < z < 0.5, and suggest that the BO effect should be interpreted as an evidence of the important role played by the cluster environment: evolution is strong in rich clusters and negligible (because it is inefficient) in poor environments.

Whether the BO effect has been confirmed or not is unclear: by studying clusters at very similar redshifts, Smail et al. (1998) and Pimbblet et al. (2002) do not find an increase of the blue fraction with redshift, although Margoniner & de Carvalho (2000) and Margoniner et al. (2001) do. Fairley et al. (2002) observed clusters at higher redshift and did not find any signature of a BO effect. Kodama & Bower (2001) and Ellingson et al. (2001) reach opposite conclusions on the existence of an excess of blue galaxies in the cluster core, the former paper using a subsample of the data used in the latter. Neither is the amplitude, when the BO effect is detected, the same in different works: Rakos & Schombert (1995) tend to find a larger blue fraction (at a fixed redshift) than BO.

The existence of the BO effect has also been criticized or simply not found when expected to show up markedly: a high blue fraction at high redshift has not been confirmed by van Dokkum et al. (2000) for the X-ray cluster MS 1054-03 at z = 0.83. Apart from all criticisms raised before 1984 and addressed in BO, Kron (1994) claimed that all the 'high' redshift clusters known at the time were somewhat extreme in their properties, and this was precisely what had allowed them to be detected. Observations of four clusters at \sim 0.4 led Oemler, Dressler & Butcher (1997) to suggest that clusters at that redshift are more exceptional objects than present-day clusters, and are actually being observed both in the act of hosting several galaxy-galaxy mergers and interactions, as well as growing by merger of smaller clumps, in agreement with a hierarchical growth of structures as described, for example, by Kauffmann (1995). The higher infall rate in the past would also favour higher blue fractions in distant clusters. Andreon et al. (1997) have made a detailed comparison of the properties of galaxies in the nearby Coma cluster and cl 0939+47 at z = 0.41. They found that the spiral population of these two clusters appears too different in spatial, colour and surface brightness distributions to be the same galaxy population observed at two different epochs. The Coma cluster is therefore unlikely to be representative of an advanced evolutionary stage of cl 0939+47, and so any comparison between the blue fraction of the two systems may be delusive. Andreon & Ettori (1999) raised two more concerns: the BO sample does not form a homogeneous sample of clusters over the studied redshift range; furthermore, optical selection of clusters is prone to produce a biased - hence inadequate - sample for studies on evolution because, at larger redshifts, it naturally favours the inclusion in the sample of clusters with a significant blue fraction. This argument is also presented by de Propris et al. (2003), who also argue that the BO effect is due to the optical selection of the galaxies: low mass galaxies with active star formation have their optical colour boosted, and these galaxies increase the cluster blue fraction.

This paper has two aims: to extend the measurement of the blue fraction to a redshift range largely not probed yet, and to review critically and discuss the analyses performed thus far by various authors in the literature. We studied three clusters selected among the best detected and possibly at high redshift cluster candidates listed in Lobo et al. (2000) and detected in the ESO Imaging Survey (EIS) data set (Nonino et al. 1999): cl 2249–3958, cl 2245–3954 and cl 0048–2942. For each one of these three clusters, we have redshift information available, confirming the presence of a galaxy overdensity at redshift 0.71, 0.66 and 0.64, respectively (Serote Roos, Lobo & Iovino 2001).

In Section 2, we present the data for the three clusters and for the control fields, and in Section 3, we detail the step-by-step description of the determination of the blue fraction of the three clusters. Results are presented in Section 4, where we also critically re-examine previous studies on the BO effect. Finally, we summarize the results and conclude in Section 5.

We adopt $\Omega_{\Lambda} = 0.7$, $\Omega_{\rm m} = 0.3$ and $H_0 = 50$ km s⁻¹ Mpc⁻¹. The H_0 value was chosen for consistency with previous works, but is largely irrelevant for the results of this paper because it cancels in the comparisons (e.g. in the difference of distance moduli or in the ratio of metric diameters).

2 THE DATA

Observations of the three clusters were performed at the 3.6-m ESO telescope at La Silla with the EFOSC2 camera, in the Bessel-Cousins *B*, *V*, *R* and *I* filters and with the Loral #40 CCD. Clusters cl 2249–3958 and cl 2245–3954 were observed in the first run (1998 October 25–27; run ID 62.O-0806), whereas cl 0048–2942 was targeted during our second run (1999 August 14–16; run ID 63.O-0689). The CCD field of view provided images covering ~5 × 5 arcmin² but a slightly different strategy (pixel binning and exposure time per filter) was adopted in each run: these and other details on the data are provided in Table 1. At z = 0.7, 3 Mpc subtends a 5-arcmin angle. r_{200} corresponds to ~0.8 Mpc, or 1.4 arcmin, for a cluster at z = 0.7 and having a velocity dispersion of 500 km s⁻¹.

Serote Roos et al. (in preparation) present a detailed description of the spectroscopic data and analysis. Shortly, targets for spectroscopy were selected from the photometric catalogueues using priority criteria based on colours and compactness. Spectroscopic observations were performed at VLT with FORS1 and FORS-2 in 1999 and 2000, at a resolution of about 500, and with a typical exposure time of 1 h. We have at least 22, 11 and seven consistent redshifts for galaxies in the fields of cl 0048–2942, cl 2245–3954 and cl 2249–3958, respectively, supporting the existence of a gravitationally bound system at ~ 0.64 , 0.66 and 0.71 in each case (see Serote Roos et al. 2001 for an early report).

Photometric data from the 3.6-m telescope have been biassubtracted, flat-fielded and fringe-corrected (when needed in the redder filters R and I) using basic IRAF routines. Then, cosmic rays were detected and flagged by using task FILTER/COSMIC in MIDAS. Pairs of images were aligned and then combined by means of imcombine in IRAF. In this step, we made full use of the masks that flagged both permanent defective pixels and cosmic rays, and took into account differences in airmass between the combined images. Finally, we kept only the sky region fully exposed in common to all filters (we note that we had applied small shifts at the telescope, of the order of 5 pixel, for optimizing the sampling especially for regions covered by defective areas of the CCD; this produced slightly different fields of view for each pointing).

Objects have been detected by SEXTRACTOR v2 (Bertin & Arnouts 1996) in double-image mode and using the *I*-band image for detection, in order to assure that colours are available for all *I*-band

Cluster name	Short name	Exposure time (s)				Seeing	Pixel size	Observed	Icompleteness
		В	V	R	Ι	(arcsec)	(arcsec)	in run	(mag)
J004831.6-294206.6	cl 0048-2942	2 × 1920	2 × 960	2×600	2×720	1.4-1.6	0.314	1999 August	23.0
J224932.1-395804.6	cl 2245-3954	2×1920	2×960	2×360	2×480	0.7-0.9	0.157	1998 October	22.8
J224513.2-395409.9	cl 2249-3958	2×1920	2×960	2×360	2×480	0.7-0.9	0.157	1998 October	23.2



Figure 1. Colour–colour diagrams for the stars. Crosses indicate their colours as given by Landolt (1992), whereas circles refer to the measurements we performed in our fields. The right-hand panel refers to our control field, whereas the left-hand panel concerns the three cluster fields. Note the good agreement between the expected and observed star loci in all cases. In the left-hand panel, there are 26 stars measured in our cluster fields, most of them falling in the crowded part of the diagram and hence not easily visible in this plot.

detected galaxies and that the resulting catalogue is actually complete in *I*, whatever the luminosity of the objects in the other filters is. For the star/galaxy classification, we used the SEXTRACTOR Class_Star index and we discarded only objects more compact than Class_Star = 0.95 in the *I* band and brighter than I = 21 mag. Fainter stars are statistically removed by using the control field, in the same way as for foreground and background galaxies (see, for example, Andreon & Cuillandre 2002). This way, we avoid rejecting compact galaxies that could be misclassified as stars because of their compactness. Magnitudes have been measured within a 4.5-arcsec aperture.

We neglected galactic absorption, which accounts for 0.01 mag at most in R - I (Schlegel, Finkbeiner & Davis 1998), the colour used in our BO analysis.

Data were calibrated by observing several Landolt (1992) standard stars. The large number of standard stars observed during the 1999 August run allowed us to compute colour terms for our system. These turned out to be very small (of the order of 0.03 at most per unit colour). We checked our photometric calibrations by comparing the star loci, in the colour–colour plane, of Landolt stars and of stars in our field of view (as, for example, in Puddu et al. 2001), and in the colour minus colour versus colour plane, in order to remove the strong correlation between colours and to emphasize systematic errors. The left-hand panel of Fig. 1 shows the good agreement between the expected and observed star loci for the stars in our cluster fields: our star sequence falls on top of the Landolt one. All colours, therefore, do not present any problems at the 0.01–0.02 mag level. All nights were photometric, with residual zero-point variations of 0.02 mag for *V*, *R*, *I* and 0.04 mag for *B*.

The resulting catalogue turns out to be complete to $m_l \sim 22.5-23$ (see Table 1), which corresponds to evolved $M_V \sim -20.0$ to -19.5 mag (see Section 3.1 for the m_I -to- M_V conversion). Completeness was estimated as in Garilli, Maccagni & Andreon (1999), Andreon et al. (2000) and Andreon & Cuillandre (2002) by looking at the magnitude of the brightest galaxies having the lowest detected central surface brightness.

As a control field, we used images of the *Hubble Deep Field*– South (HDF–S), retrieving the data from the Goddard group (see http://hires.gsfc.nasa.gov/~research/hdfs-btc/ and Palunas et al., private communication), as released on 2002 April 15. These data were taken with the Big Throughput Camera (Wittman et al. 1998) on the Blanco 4-m telescope at the Cerro Tololo Inter-american Observatory (CTIO) and calibrated in the Landolt photometric system by observing Landolt (1992) standard stars, as for our program images. The camera is a $4k \times 4k$ mosaic with large CCD gaps. The area surveyed is very large, covering 0.5 deg², large enough to get rid of the cosmic variation of galaxy counts. From that area, we excluded a few regions where obvious clusters are located, with no considerable change to the area surveyed. We produced the corresponding galaxy catalogue exactly as previously done for our cluster fields. The right-hand panel of Fig. 1 shows the locus of stars in the colour-colour plane, comparing values taken directly from the Landolt catalogue with those measured by us in the control field image. The agreement between the two loci is good, showing that the control field observations are indeed in the same photometric system of our cluster observations.

Two secondary control fields were used, in order to check that the ~0.5 deg² area of the HDF–S is a typical sky region devoid of any particular large-scale structure. The first one is the F11–22 area of the Deep Lens Survey (public images are at http://dls.belllabs.com/Publicdata/index.html), observed in *BVRI* with the Mosaic camera at the 4-m Kitt Peak National Observatory (KPNO) telescope. The field of view of the image is about 1/3 deg². The other secondary control field is the Selected Area 57 (SA57) at the North Galactic pole, whose images are presented in Andreon & Cuillandre (2002). Here, we use only the *R*-band images, which were taken with the UH8k camera (Luppino, Bredthauer & Geary 1994) at the Canada–France–Hawaii Telescope (CFHT). The field of view of that image is about 0.2 deg². Both the UH8k and the Mosaic camera are 8k × 8k devices formed by tightly packed CCDs.

Again, for these two additional control fields, we produced the corresponding galaxy catalogues with SEXTRACTOR, exactly with the same general settings as applied to the cluster images and the HDF–S control field.

The three control fields are well apart in the sky: the HDF–S is at R.A. $22^{h}34^{m}$, Dec. $-60^{\circ}37'$; the SA57 is at R.A. $13^{h}09^{m}$, Dec. $+29^{\circ}09'$; and the F11–22 field is at R.A. $0^{h}53^{m}$, Dec. $+12^{\circ}35'$, and therefore these sample three very different lines of sight. Fig. 2



Figure 2. Galaxy counts in the adopted control field (circles) and in the direction of two other control fields: the Selected Area 57 at the North Galactic pole and the F11–22 area of the Deep Lens Survey. There is no evidence for a discrepancy in galaxy counts in the HDF–S with respect to the other control fields. The areas of the three fields are, roughly, 0.50, 0.18 and 0.34 deg² for the HDF–S, SA57 and F11–22, respectively. Magnitudes were measured in an aperture of 4.5 arcsec.



Figure 3. Galaxy counts in the control field (open circles) and in the direction of the three clusters. There is a clear excess of galaxies in the line of sight of each cluster with respect to the background galaxy counts.

shows the galaxy counts for the three control fields in the R band, by adopting a 4.5-arcsec aperture magnitude. There is a good agreement among the counts measured in the three different lines of sight, showing that none of the three pointings is peculiar in galaxy density. All our three control fields are deeper than programme cluster images, and hence can be used in this work.

Fig. 3 shows the galaxy counts for the central regions of the cluster pointings and for the HDF–S control field in the *I* band, for the same aperture magnitude. There is a clear, and reassuring, excess of galaxies in the lines of sight of the clusters.

3 DETERMINATION OF THE BLUE FRACTION

BO define the fraction of blue galaxies, f_b , in the cluster as the fraction of galaxies bluer by at least 0.2 mag, in the B - V rest frame, than early-type galaxies at the cluster redshift. The galaxies have to be counted down to an absolute magnitude of $M_V = -20$ (for $H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$), within the radius R_{30} that encompasses 30 per cent of the cluster galaxies. Moreover, galaxies in the background or foreground of the cluster have to be previously removed, for example, by statistical subtraction.

The *actual* BO limiting magnitude is, at the BO high-redshift end, brighter than $M_V = -20$ mag (see de Propris et al. 2003). A brighter limiting magnitude at higher redshift is the correct choice if one wants to track the same population of galaxies at different redshifts, because of average luminosity evolution experienced by galaxies. Therefore, we adopted an evolving $M_V = -20$ mag limit, as *actually* adopted by BO. An evolving limiting magnitude has also been adopted by de Propris et al. (2003) and Ellingson et al. (2001) in their BO-style studies.

According to measurements by Lin et al. (1999) of the evolution of the characteristic luminosity of M^* in the *R* band in 0.12 < *z* < 0.55, we expect to have approximately 1-mag brightening in the *V*-band galaxy luminosities at the redshift of our clusters for our adopted cosmology, in agreement with Bruzual & Charlot (1993) models. Therefore, we adopt a 1-mag brighter limiting magnitude than the non-evolving cut.

3.1 Colour and magnitude cuts and K-corrections

In order to measure the fraction of blue galaxies, we must compute the colour cut in the observer's frame, $\Delta(m_R - m_I)$, that corresponds to the rest-frame $\Delta(B - V) = 0.2$ set by BO. To this end, we note that the (B - V) colour difference at zero redshift between an E and an Sbc is 0.34 mag (Frei & Gunn 1994), when the spectral energy distributions are taken from the Coleman, Wu & Weedman (1980) templates. The colour cut in the observer frame $\Delta(m_R - m_I)$ corresponding to the original BO definition will therefore be $\Delta(B - V) = 0.2/0.34$ times the colour difference between an E and an Sbc at the cluster redshift according to the same models. For our three clusters, we get $\Delta(m_R - m_I) = 0.26$ mag.

K-corrections, needed to perform the above calculation, have been computed according to Weinberg (1972) using the response curve of our filters (listed in the MIDAS environment) together with the quantum efficiency of the Loral CCD. We adopted as reference galaxy spectra those of Coleman et al. (1980). We checked our code, verifying that we get the same *K*-corrections as previous work in the literature (Frei & Gunn 1994). For synthetic colour measurements, we use the Vega spectrum that is included in the GISSEL96 distribution (Bruzual & Charlot 1993).

For each cluster, the median colour of the three brightest galaxies in the $(m_R - m_I)$ versus m_I colour–magnitude relation was assumed as the typical colour of early-type galaxies at the cluster redshift (and can be found in the fifth column of Table 2). Puddu et al. (2001) show that such a measure tracks the colour of the red sequence for their clusters, which spread across the range 0 < z < 0.35. Our observed colours agree well with those expected for early-type passive evolving galaxies at the redshift of our clusters.

In order to compute the corresponding M_V absolute magnitude cut, we should take into account the fact that the match between the *I* filter and the rest-frame *V* is only approximate. This implies that galaxies of identical M_V magnitudes have slightly different m_I magnitudes, with differences correlated to colour. Therefore, the cut $M_V = -20$ of BO's definition becomes, when translated to a cut

Table 2. Cluster characteristics.

Cluster name	Z	N_z	R ₃₀ (arcmin)	Red sequence $m_R - m_I \text{ (mag)}$	$\frac{\Delta(m_R - m_I)}{(\text{mag})}$	С	N _{red}	N _{all}
cl 0048-2942	0.64	22	0.8	1.12	0.26	0.4	40	50
cl 2249-3958	0.71	11	0.2	1.27	0.26	0.5	30	~ 40 (see notes)
cl 2245-3954	0.66	7	0.2	1.15	0.26	0.4	15	25

Redshifts are from Serote Roos et al. (2001).

 N_{red} is the asymptotic number of member galaxies brighter than $m_I = 22.5 \text{ mag}$ (which corresponds roughly to the evolved $M_V = -20 \text{ mag}$) integrated over the whole 'growth curve' that are red according to the BO definition.

 N_{all} refers to the same calculation but without applying any colour cut. We note that this value could not be computed directly for cl 2249–3958 because of a contaminating population in the outskirts of this cluster; it was therefore computed assuming the same global blue fraction of the cluster cl 0048–2942, which has a similar blue fraction within R_{30} and $2 \times R_{30}$ (see Section 4).



Figure 4. Colour-magnitude diagrams for the three cluster fields considering all galaxies within R_{30} . The straight horizontal line marks the colour of the red sequence, whereas the slanted line marks the evolved $M_V = -20$ mag cut. Filled symbols are statistical members, whereas open points are statistical interlopers. Small points mark galaxies falling inside $2R_{30}$.

in observed m_I magnitudes, slightly colour-dependent. Adopting *K*-corrections in the *R* and *I* filters and rest-frame (R - I) colours for the spectral templates listed in Coleman et al. (1980), we obtain the slightly slanted (almost vertical) line in Fig. 4. The galaxies to be considered for the BO effect are those to the left of this line (and for this reason fainter galaxies are not plotted). This cut has been applied to cluster and control field samples.

The slope of the colour-magnitude relation and its impact on the determination of the fraction of blue galaxies is negligible (and neglected).

3.2 Cluster radii and f_b values

In order to derive the fraction of blue galaxies in our clusters, we first need to estimate R_{30} , the radius including 30 per cent of the cluster members. We computed the radial profile, N(r), of our three clusters by counting galaxies in circular rings centred on the brightest cluster galaxy (we did take into account that our rectangular field of view truncates the outermost rings) and by statistically subtracting the background galaxy density using our control field. We used rings of increasing width outwards, in order to keep the signal-to-noise ratio (S/N) almost constant.

In order to enhance the contrast between the cluster and the background when counting N(r), we considered only galaxies brighter than $m_I = 22.5$ mag and within 0.26 mag from the colour–magnitude relation (i.e. red galaxies according to the BO definition). Needless to say, this colour selection was applied only in this step and consequences on R_{30} are quantified below. The radial profiles, N(r), for the three clusters are shown in Fig. 5, together with the expected background counts obtained from the control field for each cluster area for the same magnitude and colour cuts. In a similar way, we computed the integrated radial profiles or 'growth curves', N(< r).

The concentration index C (BO) is defined by

$$C = \log(R_{60}/R_{20}),$$

where R_{60} and R_{20} are the radii that include 60 and 20 per cent of the cluster members, respectively. The three clusters have values that classify them as 'compact' according to BO (C = 0.4–0.5; see Table 2), i.e. that qualify them as appropriate for the BO measure of the fraction of blue galaxies.

BO, working with nearer clusters, could avoid applying the colour selection we adopted for our clusters. The colour cut is especially required for cl 2249-3958 because of a blue contaminating population at large cluster-centric radii, which prevents the growth curve from converging to a constant value. Spectroscopic observations (Serote Roos et al. 2001) confirm such a contamination. We do stress that, for the other two clusters, R_{30} remains unchanged, within 10 per cent, when derived using the whole galaxy population or the red population only, therefore confirming that having chosen only red galaxies does not bias our results.

All three of our clusters have small estimated values of N_{30} and R_{30} relatively to the BO sample (see Table 2 and Fig. 6). Our smaller R_{30} radii are not due to an error in the background subtraction because the contrast between cluster and field is quite high at R_{30} . Furthermore, the background is not a free parameter: it was estimated on a wide sky area (~0.5 deg² wide), which renders the average



Figure 5. Radial profiles for the three clusters considering all red galaxies (according to the BO definition) brighter than I = 22.5 mag and previously to any background subtraction. The expected background, independently measured on the control field, is shown by the horizontal line. It differs slightly from cluster to cluster, because of differences in the considered colour range.



Figure 6. Relation between N_{30} and R_{30} (in Kpc) for the BO clusters (open circles) and our sample (filled circles). Errors on R_{30} for our sample are estimated to be of the order of 10 to 20 per cent.

value (and its variance) very well determined. We can exclude that our main control field is 'anomalous' (too dense or too sparse) or a possible mistake in the photometric calibrations, as already discussed in Section 2. We are hence confident that our estimate of R_{30} is reliable.

We can now proceed to measure $f_{\rm b}$. Some of the galaxies in each cluster line of sight will be interlopers, i.e. not physical members of the cluster but just objects projected along the line of sight, and we remove them statistically in exactly the same way as BO did.

We note that the cluster mass might alter, through lensing, the luminosity of the background population, therefore increasing the galaxy counts locally. This would artificially lead to an overestimate of the cluster counts. The change in the galaxy counts due to the lensing is a power of $2.5 \times (0.4 - \alpha)$ (Bernstein et al. 1995), where α is the slope of the galaxy counts. In the *I* band, i.e. in the filter we used for selection, we found a slope of 0.40 between 19.0 and 22.5 mag, the magnitude range in which we are interested in. Therefore, lensing has a very small effect, if at all, upon the background counts.

Table 3. Cluster blue fractions, f_{b} , measured within radius R and down to the limiting magnitude M_{V} .

Cluster name	R	$M_V{}^a$	fь	$\sigma(f_b)$
cl 0048-2942	$1 \times R_{30}$	-20.0	0.29	0.05
	$1 \times R_{30}$	-19.5	0.33	0.05
	$2 \times R_{30}$	-20.0	0.25	0.06
	$2 \times R_{30}$	-19.5	0.29	0.05
cl 2249-3958	$1 \times R_{30}$	-20.0	0.20	0.04
	$1 \times R_{30}$	-19.5	0.25	0.03
	$2 \times R_{30}$	-20.0	0.22	0.05
	$2 \times R_{30}$	-19.5	0.26	0.05
cl 2245-3954	$1 \times R_{30}$	-20.0	0.00	0.00
	$2 \times R_{30}$	-20.0	0.17	0.05

The quoted σ is half the interquartile range. For a Gaussian distribution the dispersion is 1.47 times the half interquartile range.

Errors listed in this table are as in literature, and do not take into account our discussion in Section 4.5.

^aMagnitudes corrected for evolution.

We used the control field to estimate the expected number of interlopers in R_{30} within our magnitude cut, their Poissonian fluctuations and the blue fraction in our clusters. We repeated this operation 100 times, each time performing a different extraction from the control field. We then computed the median blue fraction and the scatter around the median. Results are shown in Table 3.

The scatter computed thus far does not take into account the cosmic variance, i.e. the variance, in excess to Poissonian fluctuations, of galaxy counts. We remind the reader that no matter how well the mean background is determined, what limits the accuracy of the background subtraction is the background variance on the spatial scale where R_{30} is measured. We divided the area of the control field in cells, each cell having an area equal to πR_{30}^2 , and counted the frequency with which we observe N galaxies (with $0 < N < \infty$), therefore deriving the background variance on the R_{30} scale. For galaxies brighter than $m_l = 23$ mag, we observed variances that are 47 and 12 per cent larger than expected for a Poissonian distribution for $R_{30} = 0.8$ and 0.2 arcmin, respectively. Therefore, the Poissonian term listed in the last column of Table 3 should be multiplied by 1.47 or 1.12, depending on R_{30} .



Figure 7. Blue fraction as a function of redshift. Open circles mark the BO sample clusters with the respective error bars as published by BO; filled circles indicate our clusters. Error bars do not include the error coming from the sample representativity (see Section 4.5). The spline is the BO eye fit to the data.

4 RESULTS AND DISCUSSION

Table 2 summarizes the measured cluster characteristics: their redshift, the number of members with known redshift, N_z , R_{30} , the colour of the red sequence, the adopted colour cut, the concentration index and the (asymptotic) number of member galaxies brighter than $m_I = 22.5$ mag. The last quantity is computed twice: for red galaxies (N_{red} , eighth column) and without any colour selection (N_{all} , last column). N_{all} is comparable to the cluster richness as measured by Abell (1958): the magnitude range in which galaxies are counted is very similar and our asymptotic measurement of N_{all} is equivalent to the 3-Mpc radius adopted by Abell (1958) in order to encompass the whole cluster. The three clusters have, therefore, R = 0 to 1, R being the standard (Abell 1958) richness.

In the BO sample, the cluster richness increases with redshift (Andreon & Ettori 1999): the highest-redshift clusters are of richness R = 3 or R = 4, being by far the richest of all the sample, a result of a bias in the cluster sample available at that time (see also Kron 1994). Our three clusters are extracted from the EIS survey, which covers a small sky area. As a consequence, the probability of getting a very rich cluster is low and this is why our sample contains commonand lower-richness clusters, which turns out to be comparable in richness to the low-redshift BO sample.

Table 3 shows the blue fraction, $f_{\rm b}$, of our three clusters computed at different radii for galaxies brighter than two evolved limiting magnitudes. This table also lists Poissonian errors on the blue fraction, $\sigma(f_{\rm b})$.

While postponing to the next sections a thorough discussion of the values listed in Table 3, we compare in Fig. 7 the values derived for the BO sample and for our three clusters. The BO *extrapolated* value of the blue fraction is around 0.35 at the mean redshift of our three clusters. cl 0048–2942 has a blue fraction compatible with the extrapolation of the BO linear trend. The two other clusters have lower blue fractions. Our data, therefore, do not show any strong evidence for the presence of an increasing f_b with look-back time, in agreement with the approximately constant fraction of blue galaxies within 0.5 r_{200} found by Ellingson et al. (2001). Before drawing any final conclusion from this plot, it is instructive to take a deeper look at the analysis performed by other authors in the literature, because different, and sometimes contradictory, results have been obtained on the BO effect.

4.1 Luminosity dependence

In this subsection, we will examine how $f_{\rm b}$ changes as a function of the adopted luminosity cut-off. At low redshift, luminosity functions of different morphological types have different shapes (e.g. Binggeli, Sandage & Tammann 1988; Andreon 1998a), and therefore it is likely that the luminosity function of galaxies of different colours differs and that the fraction of blue galaxies will depend on the luminosity cut-off. On the other hand, BO have shown that, as long as this cut-off value is in the $-22 < M_V < -20$ mag range, the blue fraction does not depend on the exact value of the limiting magnitude for four of their clusters. A similar result was found by Kodama & Bower (2001) for six Einstein Medium Sensitivity Survey (EMSS) clusters and by Ellingson et al. (2001) on a sample of 15 clusters. Fairley et al. (2002) observed a more complex situation: in five out of eight clusters, the blue fraction was identical when a $M_V = -21$ or -20 mag cut-off was adopted, whereas for the three remaining cases, f_{b} was lower by 0.1 (a 1 σ effect) when the brighter cut-off was adopted. For two of our clusters, cl 0048-2942 and cl 2249-3958, we can make a measure of the BO effect both at evolved $M_V = -20.0$ and -19.5 limiting magnitudes; the f_b of these two clusters remains constant within 1σ (see Table 3). For the third cluster, cl 2245-3954, we cannot perform such a comparison because data are not deep enough.

Therefore, f_{b} does not seem to depend critically on the adopted luminosity cut-off.

4.2 Richness, radial profile and shortcomings of a unique metric aperture

In this subsection, we will examine how $f_{\rm b}$ changes as a function of the cluster radial cut-off.

We expect some radial dependence of the blue fraction because of the well-known colour segregation in clusters. At lower redshift, Fairley et al. (2002) and Kodama & Bower (2001) show that the blue fraction radial profiles differ from cluster to cluster. BO show that the $f_{\rm b}$ radial profile depends on redshift. It is therefore dangerous to assume a universal $f_{\rm b}$ radial profile for all clusters. At the higher redshifts of our sample, by taking $2 \times R_{30}$ instead of R_{30} , the blue fraction of two clusters stays constant within 1σ (Table 3), whereas the blue fraction increases for cl 2245–3954.

Margoniner & de Carvalho (2000) and Margoniner et al. (2001) (MC & M hereafter) calculate richness and blue fractions inside a fixed metric diameter of 0.7 Mpc (for $H_0 = 67$ km s⁻¹ Mpc⁻¹). This choice is fundamentally different from the one adopted by BO, whose metric radius scales with the cluster size. Because of the existence of the morphological segregation (and hence of a colour segregation), the blue fractions of clusters of different sizes are not directly comparable.

In order to remove the obvious f_b dependence on cluster size, MC & M parametrize it with a power law as a function of the cluster richness, i.e. they assume that scale, richness and colour segregation are tightly correlated, and that the correlation is the same for all clusters and does not depend on z. In the light of the results quoted above, such an assumption seems debatable.

As a test case, let us consider our two clusters cl 0048–2942 and cl 2245–3954. Measuring their f_b and richness according to MC & C prescriptions, we obtain for both clusters the same richness ($N_{MC\&C}$ ~ 30), whereas our asymptotic richness of the two clusters differs

by a factor of 2 (see Table 2). In order to obtain the f_b values derived according to the BO prescription, we need to correct the f_b derived in a 0.7 Mpc diameter by -0.16 and +0.24, respectively, whereas the correction, according to the MC & C prescription, should be the same.

The remaining cluster, cl 2249–3958, is contaminated in its outskirts by the presence of a foreground group, and therefore its MC & C richness is overestimated: the statistical correction adopted by them removes only the average background, whereas the radial profile suggested by BO helps a lot in detecting groups superposed on the cluster line of sight. This is not a rare situation, and if the measured richness is incorrect, the richness-dependent correction is incorrect too.

The Margoniner et al. (2001) claim that richer clusters tend to have lower f_b is therefore a simple restatement of the morphology– radius relation: the larger and richer the cluster is, the lower its spiral fraction (and therefore the blue fraction) in a fixed metric aperture. This is not informative at all on the dependence between richness and f_b (as defined by BO and/or Abell).

A similar fixed aperture for measuring the blue fraction has been recently used by de Propris et al. (2003) and by Goto et al. (2003), so similar concern applies to their work.

4.3 Colour cuts

A survey of the colour cuts adopted in the literature shows different choices. Kodama & Bower (2001) adopted the colour cut $\Delta(g - r) = 0.26$ to 0.40 mag, depending on redshift, whereas Ellingson et al. (2001) adopted $\Delta(g - r) = 0.21$ to 0.28 mag instead for the very same data (filters and clusters). Inspection of the Fukugita, Shimasaku & Ichikawa (1995) tables, used by Kodama & Bower (2001) in their calculations, seems to confirm the Ellingson et al. (2001) results.

Margoniner et al. claim that $\Delta(g - r) = 0.2$ mag is equivalent to $\Delta(B - V) = 0.2$ mag, and adopt the former cut for their work. However, at z = 0, $\Delta(B - V) = 0.2$ mag is the colour difference between (spectrophotometric) E and Sab galaxy types (Fukugita et al. 1995), whereas $\Delta(g - r) = 0.2$ mag is the difference in colour between E and Scd. By choosing $\Delta(g - r) = 0.2$ mag, there are spectrophotometric types that will be counted as red by Margoniner et al. but as blue by BO. Because the evolution of galaxies in clusters seems to be rather different between early- and late-type spirals (Dressler et al. 1997), MC & M are sampling a population of galaxies different from the one sampled by BO.

For our three higher-redshift clusters, by adopting $\Delta(g - r) = 0.2$ mag [instead of $\Delta(B - V) = 0.2$ mag as BO prescribe] lowers blue fractions by about 0.1, simply because spirals of intermediate spectrophotometric types are now counted as red. It is therefore clear that blue fractions computed assuming different colour cuts cannot be directly compared.

The different MC & M colour-cut choice and the different aperture adopted produce, as a final effect, a Δf_b between the two methods that can be as large as 0.4, for an average f_b of 0.2.

In a series of papers on the BO effect, Rakos and collaborators (e.g. Rakos, Schombert & Kreidl 1991; Rakos & Schombert 1995; Steindling, Brosch & Rakos 2001) adopt a different definition for the reference colour: their colour offset is measured relatively to the colour of a present-day elliptical, instead of using as reference the observed colour–magnitude relation. In this latter case, the colour is observed to evolve with redshift from the (redder) location of present-day ellipticals (e.g. Stanford et al. 1998); this is expected because the age of the universe at a given redshift is an upper limit to the age of the stars at that redshift. In other terms, their offset is not given with respect to the colour of passively evolving objects having the age of the universe at that redshift (as in the BO prescription), but with respect to 15-Gyr-old galaxies (even in a universe that may be only 7 Gyr old, for example). With the choice of Rakos et al., a cluster composed exclusively of passively evolving galaxies naturally increases its blue fraction with look-back time (i.e. z). At high enough redshift, their colour cut will eventually include in the blue fraction even the reddest galaxies at that redshift. In fact, this 'high enough' redshift corresponds to the redshift of our clusters, the reddest galaxies of which are almost 0.2 mag bluer than the colour of present-day ellipticals, thus qualifying to be classified 'blue' by the Rakos et al. criterion. While their choice is fully auto-consistent, their f_{b} cannot be directly compared with the BO one, because the two definitions are equal only at $z \sim 0$. In particular, one needs to deemphasize the Rakos et al. claim that at $z \sim 0.9$, almost all galaxies are blue; this is a consequence of their definition of 'blue', because at z = 0.9, no galaxy can be red enough to be classified as red by their criterion, simply due to the reduced age of the universe at such an epoch. For this reason, it is preferable to adopt the BO definition of the blue fraction, as we did in our analysis, which separates the bluing due to the young age of the universe from the bluing due to the BO effect itself.

Given these considerations, we believe that the large blue fraction of Rakos et al. at high redshift is no longer in contradiction with the low blue fraction at high redshift found by van Dokkum et al. (2000), and, in a more general way, their claim of a clear evidence of a BO effect should be considered with caution.

4.4 Cluster selection bias

Thus far, we have assumed that any observed sample (ours and those of other authors) is a representative sample, i.e. that selection criteria, if present, are benign. Andreon & Ettori (1999) have shown that, instead, for the BO sample this is not the case, and that the high-redshift clusters they studied are not the ancestors of their present-day clusters. In other words, and for that particular sample, one is comparing 'unripe apples to ripe oranges' in order to understand 'how fruit ripens' (Andreon & Ettori 1999). The three high-redshift clusters analysed in this paper are much poorer than the high-redshift clusters in the BO sample, thus being more similar, from this point of view, to BO's low-redshift clusters. On the other hand, they are much smaller (their R_{30} is smaller) than the large majority of the clusters in the BO sample, and so they possibly consist in another class of clusters with respect to the ones gathered in the BO sample.

The three clusters we study in the present paper have been optically selected. As explained in Aragon-Salamanca et al. (1993) and in Andreon & Ettori (1999), among all clusters of a given mass, an optical selection favours those with an unusual population of starforming galaxies. Because selection criteria and bandshift effects become more and more important as redshift increases, the optical selection might artificially increase the blue fraction as the redshift increases, hence mimicking the BO effect. With our data, we can test whether our cluster selection is biased by such an unusual population.

From the models (Bruzual & Charlot 1993), we expect that most of the cluster blue galaxies, unless they are forming stars at a significant rate, fall below the limiting magnitude of $m_I = 22$ used by Lobo et al. (2000) in the process of detecting clusters, i.e. we would expect them to be fainter than that limiting magnitude. In order to mimic a selection less biased by galaxies with an high star formation rate, we remove the blue galaxies from our three clusters and we measure

the detectability of the red population by re-inserting them in the EIS catalogue at various positions. As we do not know the membership of each individual red galaxy in the cluster line of sight, we performed 101 realizations of each background-subtracted cluster. each one using a background sample randomly extracted from our control field sample. Each cluster was then inserted 10 times (for a total of 1010 simulations per cluster) in the EIS catalogue, avoiding areas where clusters are detected. We recovered the inserted clusters 33, 50 and 65 per cent of the time for cl 2245-3954, cl 2249-3958 and cl 0048-2942, respectively. For the detection of these three clusters, and especially of the first two, the presence of an important (over the whole cluster) blue population turns out to be essential, because clusters similar to those inserted (but without a blue population) are underrepresented by a factor of $2 \left[= 3/(0.33 + 0.50 + 0.65) \right]$. Therefore, the simple fact of having selected the sample from optical photometry seems to have biased the blue fraction (towards higher values observable at higher z) by preferentially selecting, among all possible clusters, the ones with a larger blue fraction, which are more easily detected.

We remind the reader that our clusters are quite small with respect to the ones listed in BO (see Fig. 6). Their detectability would be even lower, if they were of median size, for the same richness. The small R_{30} radii of our clusters, given their richness, is a further reason to believe that the detected clusters are just the tip of the iceberg.

The same concerns may apply to other clusters selected in similar ways.

4.5 f_b errors

Last, but not least, to conclude our discussion on the effects that have a bearing on the detection of the BO effect, we should not neglect the way errors are defined.

The error associated to the computed fraction of blue galaxies strongly depends on the error definition: should it or should it not take into account the fact that we observe n cluster member galaxies subject to Poissonian fluctuations? If the aim is to measure the error on the blue fraction of the parent distribution from which the observed cluster is a member, then the answer is yes, it should. On the other hand, if one restricts oneself to measuring the error on the blue fraction of one specific cluster, then this is not needed. This fact becomes clear by reading Gehrels (1986), which we quote here. 'We consider [...] the case where an observer is measuring two different kinds of distinguishable events. It is assumed that [...] the number of events of each type [...] is distributed according to Poissonian statistics. The objective is to obtain confidence limits on the ratio of the two event rates based on the measurement of a small number of events', i.e. errors for the f_{b} fraction in our case. As Gehrels (1986) shows, the joint probability to observe n_1 red galaxies and n_2 blue galaxies is equal to the Poisson probability of observing n_1 $+ n_2$ galaxies times the binomial probability for obtaining specifically n_1 red and n_2 blue galaxies, given that the combined number of galaxies observed is $n_1 + n_2$. The binomial probability alone, adopted, for example, by Metevier, Romer & Ulmer (2000),¹ gives errors that are somewhat conservative for small $n_1 + n_2$.

¹ There is a typo in the formula quoting the variance of binomial distribution given in Metevier et al. (2000) and in 'Notes on statistics for physicists, revised' by J. Orear, the latter distributed by the NASA/IPAC Extragalactic Database (NED). The correct formula for the square root of the variance of a binomial distribution is $\sigma(p) = \sqrt{p(1-p)/N}$, where *N* is the number of trials and *p* is the probability of success.



Figure 8. Blue fraction as a function of redshift, including errors on the sample representativity. Line, open and filled circles are defined as in Fig. 7.

Such a discussion would be useless if the error coming from other sources (the background subtraction, for example) dominated the overall error budget. This indeed is not the case for the BO clusters, for which the full error (including everything) listed in BO is often smaller than the Gehrels error alone.

If one adopts the Gehrels (1986) error formula, then the error on the blue fraction of cl 2245–3954 becomes non-zero, as it should be.

To summarize, BO and – given the absence of similar remarks in the literature – most of the literature seem to quote errors by assuming that 'repetition of experiment' means 're-observing the same cluster', i.e. observing exactly the same number of galaxies. Here, instead, we claim that errors should be quoted as if 'repetition of experiment' means 'observation of any cluster drawn from the same parent population'.

The underestimate of the error implies that any existing trend of the blue fraction with *z* has been emphasized more than the statistics allow. Fig. 8 shows the blue fraction as a function of redshift, plotted with error bars that now take into account the binomial term, the Poissonian term and over-Poissonian background subtraction errors. For the BO clusters, we simply add (quadratically) to the error quoted by BO the ones computed according to Gehrels (1986) because of lack of information. The figure also includes our three clusters, shown as solid dots. Given the large error bars, the data can, after all, be described by a constant $f_{\rm b}$.

5 SUMMARY

Through the observations of three clusters at $z \sim 0.7$, of richness comparable to the low-redshift sample of BO, we have determined their fraction of blue galaxies, $f_{\rm b}$. According to the standard analyses (those presented in BO, of widespread acceptance and shown in Fig. 7), two clusters have a low blue fraction for their redshifts, and the fraction of the third one is compatible with the expected value.

We studied the impact of relaxing each one of the BO criteria in the calculation of the blue fraction.

(i) The exact choice of the luminosity cut is not critical, provided it differs by 1 mag or less.

(ii) Adopting a unique metric radii for all clusters, regardless of them being large or small, and eventually correcting for the richness dependence, as is sometimes performed in the literature, is not informative on the BO effect because of the contamination by the colour segregation in clusters. The $f_{\rm b}$ measured within a metric aperture is therefore informative about something different from the BO effect.

(iii) The colour cut is also important. In some cases, we are unable to reproduce colour cuts of other authors; in other cases, we show that the adopted colour cuts differ from those defined in BO. Because galaxies of different colours have probably different star formation histories, the comparison of blue fractions derived using different cuts is not straightforward. For our three clusters, the blue fraction decreases by 0.1 when adopting the more liberal $\Delta(g - r) = 0.2$ colour cut, instead of the standard $\Delta(B - V) = 0.2$.

(iv) The adoption of a correct reference colour is a critical point. If the reference colour does not change appropriately with look-back time, then the measurements of the BO effect are contaminated by the bluing due to the younger galaxy ages at higher redshift. Two of our clusters at $z \sim 0.7$ have a huge blue fraction according to the definition of Rakos et al., simply because the universe is so young at $z \sim 0.7$ that no stellar population can be red enough to be called red according to their criterion. However, this has nothing to do with the BO effect.

Finally, a re-analysis of the error computations usually performed in the literature shows that the f_b errors quoted by BO (and likely by other authors) underestimate the real errors. If we plot the original BO data together with our three high-redshift clusters, with the newly determined error bars, we cannot exclude a constant f_b .

Therefore we conclude the following.

(i) The correct comparison of BO effect determinations reported by different authors is a task as difficult as performing the measurement itself, and both should be done with extreme carefulness.

(ii) Twenty years after the original intuition by BO, we are still in the process of ascertaining the reality of the BO effect.

In a future paper, we will present a BO-style analysis for an Xray selected sample of clusters collected by the *XMM*–LSS project (Pierre et al. 2004), hence overcoming the biases of the cluster optical selection.

ACKNOWLEDGMENTS

We warmly thank Reinaldo de Carvalho for numerous discussions on the subject. We acknowledge partial support from project ESO/PRO/15130/1999 (FCT, Portugal). This paper is based on observations obtained at the 3.6-m ESO, Canada–France–Hawaii, Cerro–Tololo and Kitt Peak telescopes.

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