Max–Planck–Institut für Astrophysik





#### Lectures, INAF-Osservatorio Astronomico di Brera 19. & 20. November 2013

#### The Violent Deaths of Massive Stars

## **Unravelling the Explosion Mechanisms**

**Connecting Theory to Observations** 

Hans-Thomas Janka (Max-Planck-Institut für Astrophysik, Garching, Germany)

# Things that blow up

supernovae

- CO white dwarf → Type Ia SN, E≈1Bethe
- MgNeO WD, accretion → AIC, faint SN
- "SAGB" star (AGB, then SN) → EC SN
- "normal" SN (Fe core collapse) → Type II SN
- WR star (Fe CC) → Type lb/c
- "Collapsar", GRB 
   broad line Ib/a SN, "hypernova"
- Pulsational pair SN → multiple, nested Type I/II SN
- Very massive stars → pair SN,≲100B (1B=10<sup>51</sup> erg)
- Very massive collapsar → IMBH, SN, hard transient
- GR He instability → >100 B SN+SMBH, or 10,000 B
- Supermassive stars → ≥100000 B SN or SMBH



MASS

#### **Final Stages of Massive Star Evolution**



## Contents

#### Lecture I:

- Supernovae: classification and phenomenology
- Basics of stellar evolution & death scenarios in overview
- White dwarfs and thermonuclear supernovae

#### Lecture II:

- Gravitational (core-collapse) supernovae: evolution stages
- SN modeling: some technical aspects
- Status of 2D and 3D SN modeling

#### Lecture III:

- Supernova models: Predictions of observable signals
- Neutron stars: birth and death
- Black holes and gamma-ray bursts: Sources of heavy elements

SN 1994d

# Thermonuclear (Type Ia) Supernovae

Standard candles for measuring the universe

### Type Ia Supernovae



**Recalibration with Phillips relation** 

#### Exploding accreting white dwarfs in binary systems

"standard candles"



### Type Ia SNe and Cosmology



#### **Observational Constraints of Cosmic Parameters**



WMAP results from Spergel et al. 2003

REFLEX results from Schuecker et al. 2003 (three weeks before WMAP publication)

#### How does the model work?



Temperature: a few 10<sup>9</sup> K

Radii: a few 1000 km

Explosion energy: Fusion C+C, C+O,  $O+O \rightarrow "Fe"$ 

Laminar burning velocity:  $U_L \sim 100 \text{ km/s} << U_s$ 

Too little is burned!

What is the mode of nuclear burning in SNe Ia?

"Detonation":

(Super-) Sonic front; heating to ignition by a shock wave. "Deflagration": Subsonic front; heating to ignition by heat diffusion. Strong Si-lines at maximum light: Pure detonations are excluded! (But possibly at lower densities???)

#### The physics of turbulent combustion

Everydays experience: Turbulence increases the burning velocity. In a star: Reynoldsnumber ~ 10<sup>14</sup> ! In the limit of strong turbulence:  $U_{R} \sim V_{T}$ ! Physics of thermonuclear burning is very similar to premixed chemical flames.



#### SN Ia Explosion



F. Röpke, W. Hillebrandt (2005)

[cm]

2e+09

- Constanting



#### Type Ia Supernovae – Achievements and Insights

- Deflagration models explode.
- Explosion energy ~0.8\*10<sup>51</sup> ergs (a bit low), too much unburned C+O left.
- Need of deflagration to detonation transition.
- Explosion energy and produced Ni depends on ignition conditions but not on initial composition.
- Brightness depends on amount of Ni produced, but only weakly on C+O composition.

#### **Deflagration-Detonation Transition**



#### Single Degenerate vs. Double Degenerate Scenario

## Interaction with companion star can "revive" dead, old white dwarf.



SD scenario: Gas accretion from close companion



DD scenario: Merging of two white dwarfs

WD Mergers



(Pakmor et al., Nature, 2010)

### Type Ia Supernovae – Open Questions and Problems

- Probably there exist different types of progenitors. Progenitor systems have not been observed yet !
- Double-degenerate scenario seems favored over single-degenerate scenarios because of
  - delay-time distribution
  - X-ray luminosity of galaxies too low for SD systems
  - WD merger rates can account for SNIa rate, accreting MCh Wds too rare
  - SNIa environments?
- How does thermonuclear ignition of white dwarf start?
- Where and how does transition from deflagration to detonation occur?
- What is the reason for the Phillips relation? Are there any systematic uncertainties?

# Type Ia Supernovae

# Exploding accreting or merging white dwarfs in binary systems

#### Used as "standard candles" for measuring distances in the Universe

# "Ordinary" Supernovae

Gravitational collapse and explosions of stars with  $8 M_{sun} < M_{*} < 100 M_{sun}$ 



Stellar Core Collapse and Explosion

# Evolved massive star prior to its collapse:

Star develops onion-shell structure in sequence of nuclear burning stages over millions of years



(layers not drawn to scale)

# Evolved massive star prior to its collapse:

Star develops onion-shell structure in sequence of nuclear burning stages over millions of years



(layers not drawn to scale)

#### Gravitational instability of the stellar core:

Stellar iron core begins collapse when it reaches a mass near the critical Chandrasekhar mass limit

Collapse

becomes dynamical because of electron captures and photodisintegration of Fe-group nuclei 0

Si

Fe

# Core bounce at nuclear density:

Si

Accretion

Fe

Inner core bounces when nuclear matter density is reached and incompressibility increases

Shock wave forms

Shock wave

#### Proto-neutron star



#### Shock "revival":

Si

**n**, '

Si

D

**Accretion** 

Stalled shock wave must receive energy to start reexpansion against ram pressure of infalling stellar core.

Shock can receive fresh energy from neutrinos!

Shock wave

#### Proto-neutron star

#### Explosion:

Shock wave expands into outer stellar layers, heats and ejects them.

Creation of radioactive nickel in shock-heated Si-layer.

n, p

n, p, α

Shock wave

Proto-neutron star (PNS)

#### Nucleosynthesis during the explosion:

Ni

n, p,

(Z<sub>k</sub>,

n, p

α,

Shock-heated and neutrinoheated outflows are sites for element formation

Shock wave

Neutrinodriven "wind But: Is neutrino heating strong enough to initiate the explosion against the ram pressure of the collapsing stellar shells?

Most sophisticated, self-consistent numerical simulations of the explosion mechanism in 2D and 3D are necessary!



#### **Supernova Scales**



$$\frac{\partial\sqrt{\gamma}\rho W}{\partial t} + \frac{\partial\sqrt{-g}\rho W\hat{v}^{i}}{\partial x^{i}} = 0,$$
(2.5)
$$\frac{\partial\sqrt{\gamma}\rho hW^{2}v_{j}}{\partial t} + \frac{\partial\sqrt{-g}\left(\rho hW^{2}v_{j}\hat{v}^{i} + \delta^{i}_{j}P\right)}{\partial x^{i}} = \frac{1}{2}\sqrt{-g}T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x^{j}} + \left(\frac{\partial\sqrt{\gamma}S_{j}}{\partial t}\right)_{C},$$
(2.6)
$$\frac{\partial\sqrt{\gamma}\tau}{\partial t} + \frac{\partial\sqrt{-g}\left(\tau\hat{v}^{i} + Pv^{i}\right)}{\partial x^{i}} = \alpha\sqrt{-g}\left(T^{\mu0}\frac{\partial\ln\alpha}{\partial x^{\mu}} - T^{\mu\nu}\Gamma^{0}_{\mu\nu}\right) + \left(\frac{\partial\sqrt{\gamma}\tau}{\partial t}\right)_{C}.$$
(2.7)
$$\frac{\partial\sqrt{\gamma}\rho WY_{e}}{\partial t} + \frac{\partial\sqrt{-g}\rho WY_{e}\hat{v}^{i}}{\partial x^{i}} = \left(\frac{\partial\sqrt{\gamma}\rho WY_{e}}{\partial t}\right)_{C},$$
(2.8)
$$\frac{\partial\sqrt{\gamma}\rho WX_{k}}{\partial t} + \frac{\partial\sqrt{-g}\rho WX_{k}\hat{v}^{i}}{\partial x^{i}} = 0.$$
(2.9)

#### General-Relativistic 2D Supernova Models of the Garching Group

(Müller B., PhD Thesis (2009); Müller et al., ApJS, (2010))

#### **GR hydrodynamics (CoCoNuT)**

$$\hat{\Delta}\Phi = -2\pi\phi^5 \left(E + \frac{K_{ij}K^{ij}}{16\pi}\right), \qquad (2.10)$$

**CFC metric equations** 

$$\hat{\Delta}(\alpha\Phi) = 2\pi\alpha\phi^5 \left(E + 2S + \frac{7K_{ij}K^{ij}}{16\pi}\right), \qquad (2.11)$$

$$\hat{\Delta}\beta^{i} = 16\pi\alpha\phi^{4}S^{i} + 2\phi^{10}K^{ij}\hat{\nabla}_{j}\left(\frac{\alpha}{\Phi^{6}}\right) - \frac{1}{3}\hat{\nabla}^{i}\hat{\nabla}_{j}\beta^{j}, \qquad (2.12)$$

$$\frac{\partial W\left(\hat{J}+v_{r}\hat{H}\right)}{\partial t}+\frac{\partial}{\partial r}\left[\left(W\frac{\alpha}{\phi^{2}}-\beta_{r}v_{r}\right)\hat{H}+\left(Wv_{r}\frac{\alpha}{\phi^{2}}-\beta_{r}\right)\hat{J}\right]-(2.28)\right]}{\frac{\partial}{\partial \varepsilon}\left\{W\varepsilon\hat{J}\left[\frac{1}{r}\left(\beta_{r}-\frac{\alpha v_{r}}{\phi^{2}}\right)+2\left(\beta_{r}-\frac{\alpha v_{r}}{\phi^{2}}\right)\frac{\partial \ln \phi}{\partial r}-2\frac{\partial \ln \phi}{\partial t}\right]+W\varepsilon\hat{H}\left[v_{r}\left(\frac{\partial \beta_{r}\phi^{2}}{\partial r}-2\frac{\partial \ln \phi}{\partial t}\right)-\frac{\alpha}{\phi^{2}}\frac{\partial \ln \phi}{\partial r}+\alpha W^{2}\left(\beta_{r}\frac{\partial v_{r}}{\partial r}-\frac{\partial v_{r}}{\partial t}\right)\right]-(2.28)\right]}{\frac{\partial}{\partial \varepsilon}\left\{W\varepsilon\hat{H}\left[\frac{1}{r}\left(\beta_{r}-\frac{\alpha v_{r}}{\phi^{2}}\right)+2\left(\beta_{r}-\frac{\alpha v_{r}}{\phi^{2}}\right)\frac{\partial \ln \phi}{\partial r}-2\frac{\partial \ln \phi}{\partial t}\right]+W\varepsilon\hat{H}\left[v_{r}\left(\frac{\partial \beta_{r}\phi^{2}}{\partial r}-2\frac{\partial \ln \phi}{\partial t}\right)-\frac{\alpha}{\phi^{2}}\frac{\partial \ln \phi}{\partial r}+\alpha W^{2}\left(\beta_{r}\frac{\partial v_{r}}{\partial r}-\frac{\partial v_{r}}{\partial t}\right)\right]-(2.28)\right]}{\frac{\partial}{\partial \varepsilon}\left\{W\varepsilon\hat{H}\left[\frac{1}{r}\left(\beta_{r}-\frac{\alpha v_{r}}{\phi^{2}}\right)+2\left(\beta_{r}-\frac{\alpha v_{r}}{\phi^{2}}\right)\frac{\partial \ln \phi}{\partial r}-2\frac{\partial \ln \phi}{\partial t}\right]+W\varepsilon\hat{H}\left[v_{r}\left(\frac{\partial \beta_{r}\phi^{2}}{\partial r}-2\frac{\partial \ln \phi}{\partial t}\right)-\frac{\alpha}{\phi^{2}}\frac{\partial \ln \phi}{\partial r}+\alpha W^{2}\left(\beta_{r}\frac{\partial v_{r}}{\partial r}-\frac{\partial v_{r}}{\partial t}\right)\right]\right]-(2.28)}{\frac{\partial}{\partial \varepsilon}\left\{W\varepsilon\hat{H}\left[\frac{1}{r}\left(\beta_{r}-\frac{\alpha v_{r}}{\phi^{2}}\right)+2\left(\beta_{r}-\frac{\alpha v_{r}}{\phi^{2}}\right)\frac{\partial \ln \phi}{\partial r}-2\frac{\partial \ln \phi}{\partial t}\right]+W\varepsilon\hat{H}\left[v_{r}\left(\frac{\partial \beta_{r}\phi^{2}}{\partial r}-2\frac{\partial \ln \phi}{\partial t}\right)-\frac{\alpha}{\phi^{2}}\frac{\partial \ln \phi}{\partial r}+2\frac{\partial \ln \phi}{\partial t}\right]\right]+W\varepsilon\hat{H}\left[\frac{\partial}{\partial \varepsilon}\left(W\varepsilon\hat{H}\left[\frac{1}{r}\left(\beta_{r}-\frac{\alpha v_{r}}{\partial r}\right)+2\left(\beta_{r}-\frac{\alpha v_{r}}{\partial r}\right)\frac{\partial \ln \phi}{\partial t}-2\frac{\partial \ln \phi}{\partial t}\right]\right]+W\varepsilon\hat{H}\left[\frac{\partial}{\partial \varepsilon}\left(W\varepsilon\hat{H}\left[\frac{1}{r}\left(\beta_{r}-\frac{\alpha v_{r}}{\partial r}\right)+2\left(\beta_{r}-\frac{\alpha v_{r}}{\partial r}\right)\frac{\partial \omega v_{r}}{\partial t}\right]\right]+W\varepsilon\hat{H}\left[\frac{\partial}{\partial \varepsilon}\left(W\varepsilon\hat{H}\left[\frac{\partial w}{\partial r}-2\frac{\partial \ln \phi}{\partial t}\right)-\frac{\alpha}{\phi^{2}}\frac{\partial \ln \phi}{\partial r}+2\frac{\partial \ln \phi}{\partial t}\right]\right]+W\varepsilon\hat{H}\left[\frac{\partial w}{\partial \varepsilon}\left(W\varepsilon\hat{H}\left[\frac{\partial w}{\partial r}+2\frac{\partial w}{\partial r}\right)+W\varepsilon\hat{H}\left[\frac{\partial w}{\partial \tau}\left(W\varepsilon\hat{H}\left(Wv_{r}\frac{\partial w}{\partial r}+2\frac{\partial w}{\partial \tau}\right)\frac{\partial w}{\partial \tau}\right)\right]\right]+W\varepsilon\hat{H}\left[\frac{\partial w}{\partial \tau}\left(W\varepsilon\hat{H}\left(Wv_{r}\frac{\partial w}{\partial r}+2\frac{\partial w}{\partial \tau}\right)+W\varepsilon\hat{H}\left(Wv_{r}\frac{\partial w}{\partial \tau}+2\frac{\partial w}{\partial \tau}\right)\right]+W\varepsilon\hat{H}\left[\frac{\partial w}{\partial \tau}\left(W\varepsilon\hat{H}\left(Wv_{r}\frac{\partial w}{\partial \tau}+2\frac{\partial w}{\partial \tau}\right)+W\varepsilon\hat{H}\left(Wv_{r}\frac{\partial w}{\partial \tau}+2\frac{\partial w}{\partial \tau}\right)\right]+W\varepsilon\hat{H}\left[\frac{\partial w}{\partial \tau}\left(W\varepsilon\hat{H}\left(Wv_{r}\frac{\partial w}{\partial \tau}+2\frac{\partial w}{\partial \tau}\right)+W\varepsilon\hat{H}\left(Wv_{r}\frac{\partial w}{\partial \tau}+2\frac{\partial w}{\partial \tau}\right)\right]+W\varepsilon\hat{H}\left[\frac{\partial w}{\partial \tau}\left(W\varepsilon\hat{H}\left(Wv_{r}\frac{\partial w}{\partial \tau}+2\frac{\partial w}{\partial \tau}\right)+W\varepsilon\hat{H}\left(Wv_{r}\frac{\partial w}{\partial \tau}+2\frac{\partial w}{\partial \tau}\right)\right]+W\varepsilon\hat{H}\left[\frac{\partial w}{\partial \tau}\left(W\varepsilon\hat{H}\left(Wv_{r}\frac{\partial w}{\partial \tau}+2\frac{\partial w}{\partial \tau}\right)+W\varepsilon\hat{H}\left(Wv_{r}\frac{\partial w}$$

### Neutrino Reactions in Supernovae

Beta processes:

Neutrino scattering:

Thermal pair processes:

Neutrino-neutrino reactions:

•  $e^- + p \rightleftharpoons n + v_e$ 

• 
$$e^+ + n \rightleftharpoons p + \bar{v}_e$$

- $e^- + A \rightleftharpoons v_e + A^*$
- $v + n, p \rightleftharpoons v + n, p$
- $\nu + A \rightleftharpoons \nu + A$
- $\nu + e^{\pm} \rightleftharpoons \nu + e^{\pm}$
- $N+N \rightleftharpoons N+N+\nu+\bar{\nu}$

• 
$$e^+ + e^- \rightleftharpoons v + \bar{v}$$

- $v_x + v_e, \bar{v}_e \rightleftharpoons v_x + v_e, \bar{v}_e$  $(v_x = v_\mu, \bar{v}_\mu, v_\tau, \text{ or } \bar{v}_\tau)$
- $v_e + \bar{v}_e \rightleftharpoons v_{\mu,\tau} + \bar{v}_{\mu,\tau}$

# The Curse and Challenge of the Dimensions

Boltzmann equation determines neutrino distribution function in 6D phase space and time  $f(r, \theta, \phi, \Theta, \Phi, \epsilon, t)$ 

Integration over 3D momentum space yields source terms for hydrodynamics  $Q(r, \theta, \phi, t), \dot{Y}_e(r, \theta, \phi, t)$ 

#### **Solution approach**

- **3D** hydro + **6D** direct discretization of Boltzmann Eq. (code development by Sumiyoshi & Yamada '12)
- **3D** hydro + two-moment closure of Boltzmann Eq. (next feasible step to full 3D; O. Just et al. 2013)

#### • **3D** hydro + "**ray-by-ray-plus**" variable Eddington factor method (method used at MPA/Garching)

• **2D** hydro + "**ray-by-ray-plus**" variable Eddington factor method (method used at MPA/Garching)



#### **Required resources**

- $\geq$  10–100 PFlops/s (sustained!)
- $\geq$  1–10 Pflops/s, TBytes
- $\geq 0.1-1$  PFlops/s, Tbytes
- $\geq 0.1-1$  Tflops/s, < 1 TByte

Explosion Mechanism: Most Sophisticated Current Models

# Explosions of $M_{star} \sim 8-10 M_{sun}$ Stars

## SN Progenitors: Core density profiles



## **SN** Simulations:

"Electron-capture supernovae" or "ONeMg core supernovae"



Kitaura et al., A&A 450 (2006) 345; Janka et al., A&A 485 (2008) 199

Convection is not necessary for launching explosion but occurs in NS and in neutrino-heating layer



- No prompt explosion !
- Mass ejection by "neutrino-driven wind" (like Mayle & Wilson 1988 and similar to AIC of WDs; see Woosley & Baron 1992, Fryer et al. 1999; Dessart et al. 2006)
- Explosion develops in similar way for soft nuclear EoS (i.e. compact PNS) and stiff EoS (less compact PNS)



## 2D SN Simulations: $M_{eter} \sim 8...10 M_{eter}$

**Convection** leads to slight increase of explosion energy, causes explosion asymmetries, and ejects n-rich matter!



t = 0.097 s after core bounce





Y.



Janka et al. (2008), Wanajo et al. (2011), Groote et al. (in preparation)



t = 0.262 s after core bounce

# 2D SN Simulations: $M_{star} \sim 8...10 M_{sun}$



CRAB Nebula with pulsar, remnant of Supernova 1054

#### Explosion properties:

 $E_{exp} \sim 10^{50} \text{ erg} = 0.1 \text{ bethe}$ M<sub>Ni</sub> ~ 0.003 M<sub>sun</sub>

Low explosion energy and ejecta composition (little Ni, C, O) of ONeMg core explosion are compatible with CRAB (SN1054)

> (Nomoto et al., Nature, 1982; Hillebrandt, A&A, 1982)

Might also explain other lowluminosity supernovae (e.g. SN1997D, 2008S, 2008HA)

# Explosions of Stars with M<sub>star</sub> >10 M<sub>sun</sub>

#### **Relativistic 2D CCSN Explosion Models**





Violent, quasi-periodic, large-amplitude shock oscillations (by SASI) can lead to runaway and onset of explosion.

They also produce variations of neutrino emission and gravitational-wave signal.



#### **SASI: Standing Accretion Shock Instability**

Nonradial, oscillatory shockdeformation modes (mainly I = 1, 2) caused by an amplifying cycle of advective-acoustic perturbations.

Blondin et al., ApJ (2003), Foglizzo (2002), Foglizzo et al. (2006,2007)





**Fig. 1.** Schematic view of the advective-acoustic cycle between the shock at  $R_s$  (thick solid line) and the coupling radius,  $R_c$  (thick dashed line), in the linear regime, shown for the case where the oscillation period of the shock ( $\tau_{osc}$ ) equals the cycle duration,  $\tau_{aac}$ . Flow lines carrying vorticity perturbations downwards are drawn as solid lines, and the pressure feedback corresponds to dotted lines with arrows. In the gray shaded area around  $R_c$  the flow is decelerated strongly.

$$\tau_{\rm aac}^{\nabla} \equiv \int_{R_{\nabla}}^{R_{\rm sh}} \frac{\mathrm{d}r}{|v|} + \int_{R_{\nabla}}^{R_{\rm sh}} \frac{\mathrm{d}r}{|v|}$$

Scheck et al., A&A 447, 931 (2008)

## **2D SN Explosion Models**

- Basic confirmation of the neutrino-driven mechanism
- Confirmation of reduction of the critical neutrino luminosity for explosions in self-consistent 2D treatments compared to 1D

Explosions in 2D simulations were also obtained recently by Suwa et al. (2010, 2012), Takiwaki et al. (2013) and Bruenn et al. (ApJL, 2013). BUT: There are important quantitative differences between all models.

Many numerical aspects, in particular also neutrino transport treatment, are different; code comparisons are needed!

# Challenge and Goal: 3D

- 2D explosions seem to be "marginal", at least for some progenitor models and in some of the most sophisticated simulations.
- Nature is three dimensional, but 2D models impose the constraint of axisymmetry (--> toroidal structures).
- Turbulent cascade in 3D transports energy from large to small scales, which is opposite to 2D.
- Does SASI also occur in 3D?
- 3D models are needed to confirm explosion mechanism suggested by 2D simulations!

## 2D vs. 3D Morphology



#### (Images from Markus Rampp, RZG)

# Computing Requirements for 2D & 3D Supernova Modeling

**Time-dependent simulations:**  $t \sim 1$  second,  $\sim 10^6$  time steps!

CPU-time requirements for one model run:

★ In 2D with 600 radial zones, 1 degree lateral resolution:

~  $3*10^{18}$  Flops, need ~ $10^{6}$  processor-core hours.

★ In 3D with 600 radial zones, 1.5 degrees angular resolution:

~  $3*10^{20}$  Flops, need ~ $10^{8}$  processor-core hours.

GARCHING





John von Neumann Institut für Computing





## **3D Supernova Simulations**

# EU PRACE and GAUSS Centre grants of ~360 million core hours allow us to do the first 3D simulations on 16.000 cores.











#### SuperMUC Petascale System





### **3D Core-Collapse Models**

27 M<sub>sun</sub> progenitor (WHW 2002)

27 M<sub>sun</sub> SN model with neutrino transport develops **spiral SASI** as seen in idealized, adiabatic simulations by Blondin & Mezzacappa (Nature 2007)

> F. Hanke et al., arXiv:1303.6269



#### **3D Core-Collapse Models**



F. Hanke et al., arXiv:1303.6269

27 M<sub>sun</sub> progenitor (WHW 2002): Spiral mode axis

## Laboratory Astrophysics

"SWASI" Instability as an analogue of SASI in the supernova core Foglizzo et al., PRL 108 (2012) 051103





#### **Constraint of experiment: No convective activity**







## **Summary**

- 2D models with relativistic effects (2D GR and approximate GR) yield explosions for "soft" EoSs, but explosion energy may tend to be low.
- 3D modeling has only begun. No clear picture of 3D effects yet.
   But SASI can dominate (certain phases) also in 3D models!
- 3D models do not yet show explosions, but **still need higher resolution** for convergence.
- Progenitors are 1D, but shell structure and initial progenitor-core asymmetries can affect onset of explosion (cf. Couch & Ott, arXiv:1309.2632)! How important is slow rotation for SASI growth?
- Missing physics ?????